Maxwell's Equations and the Absolute Lorentz Transformation

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Abstract: This note supplements Chapter 8 of my thesis that studies Maxwell's equations under the Absolute Lorentz Transformation (A.L.T.), and it compares in greater detail the fields transformed under the A.L.T. with those under the L.T. The general covariance of Maxwell's equations is reviewed, and it is noted that in the case of flat spacetime this includes the A.L.T. The d'Alembertian equation under the A.L.T. is given for the vector potential in the Landau gauge which is shown to be invariant under linear transformations. It is also pointed out that the trajectory of a particle will be the same with the L.T. or the A.L.T., except that the two sets of clocks will record different travel times; although they will agree for a round-trip journey.

This paper is a supplementary background to Chapter 8 of my thesis that will hopefully make it clear that certainly Maxwell's equations hold under the Absolute Lorentz Transformation (A.L.T.) as well as further clarify how the electromagnetic fields transformed under the A.L.T. compare with those transformed under the Lorentz Transformation (L.T.). First of all, it should be kept in mind that, following Einstein's principle of general covariance, when Maxwell's equations are written in generally covariant form they hold in *all* coordinate systems, not just under the L.T. or the A.L.T. Unfortunately, for physicists and engineers only exposed to special relativity, and who therefore think solely in terms of the L.T., this more general result comes as something of a shocker! But of course one has to consider carefully what are the measured quantities when one employs these alternative transformations, and as regards the A.L.T. and the linear local time transformation, this is done in Chapter 8. But before going into this in detail, I wish to review the generally covariant form of Maxwell's equations.

As in special relativity, one introduces a second-rank antisymmetric tensor for the electromagnetic field, $F_{\mu\nu} = -F_{\nu\mu}$, with $\mu, \nu = 0, 1, 2, 3$, and for simplicity, $c = 1$, and further on below I will occasionally set $x^0 = t$, $x^1 = x$, $x^2 = y$, $x^3 = z$. One can readily show that because of the asymmetry, $F_{\mu\nu}$ has only six linearly independent components given by F_{0i} with $(i = 1, 2, 3)$ corresponding to the three components of the electromagnetic field, and to a suitable set of the components of the F_{ij} corresponding to the three components of the magnetic field. Note that different authors have different conventions so that, *e.g.*, F_{0i} might for

some correspond to the positive components of the electric field, while for others it might correspond to the negative component. It is also sometimes more convenient to work with the contravariant form of the electromagnetic tensor which is given by $F^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta}$, and summation over the repeated indices α, β is understood. The second rank symmetric tensor $g^{\mu\nu}$ is the contravariant form of the metric tensor and is also its inverse, so that $g^{\mu\alpha} g_{\nu\alpha} = \delta^\mu_\nu$, where the latter is the identity matrix, with $\delta^\mu_\nu = 0$, $\mu \neq \nu$, and $\delta^\mu_\nu = 1$, $\mu = \nu$, no sum. One can show that $F^{\mu\nu} = - F^{\nu\mu}$, as is true for the covariant form of the tensor used above. (Note that the term *covariant* is used in two different ways: sometimes it refers to putting equations in tensor form, and sometimes it refers to where the tensorial indices are located, hence with covariant forms, the indices are below, and with contravariant forms, the indices are above, and for second rank tensors or higher, there are mixed forms.) The proof of the asymmetry of $F^{\mu\nu}$ follows from the asymmetry of $F_{\mu\nu}$ and the symmetry of $g^{\mu\nu}$. One has $F^{\nu\mu} = g^{\nu\beta} g^{\mu\alpha} F_{\beta\alpha} = - g^{\nu\beta} g^{\mu\alpha} F_{\alpha\beta} = - F^{\mu\nu}$.

It is shown in textbooks dealing with special relativity and electromagnetism that Maxwell's equations in all Lorentz invariant systems take the following form with partial derivatives replaced by a comma, thus $\frac{\partial f}{\partial x}$ is replaced by $f,_x$, so that one has, see (8.1) in the thesis,

$$F_{\mu\nu,\lambda} + F_{\lambda\mu,\nu} + F_{\nu\lambda,\mu} = 0\,, \tag{1}$$

$$F^{\mu\nu}_{,\nu} = j^\mu. \tag{2}$$

To put these equations in generally covariant form, one replaces the commas by semicolons that indicate covariant derivatives, so that the above equations become

$$F_{\mu\nu;\lambda} + F_{\lambda\mu;\nu} + F_{\nu\lambda;\mu} = 0\,, \tag{3}$$

$$F^{\mu\nu}_{;\nu} = j^\mu\,. \tag{4}$$

Now a remarkable simplification occurs because of the asymmetry of the $F_{\mu\nu}$, and the symmetry of the Christoffel symbols that are involved in the covariant derivatives that are in (3). The first term can be written, $F_{\mu\nu;\lambda} = F_{\mu\nu,\lambda} - \Gamma^\alpha_{\mu\lambda} F_{\alpha\nu} - \Gamma^\beta_{\nu\lambda} F_{\mu\beta}$, and similar expressions for the other two covariant derivatives. Well, one finds that all the terms involving the Christoffel symbols cancel, so the covariant derivatives can all be replaced by partial derivatives, or commas, so the equation takes the same form as (1). Although in our work, *all* the Christoffel symbols vanish, since they involve partial derivatives of the metric tensor, and we are working within flat spacetime with Cartesian spacetime coordinates,

for which all the metric coefficients are constants, and hence their partial derivatives vanish, nevertheless it is interesting to see that the first equation of Chapter 8 and (1) above holds more generally than in special relativity: *it is true even in general relativity in arbitrary systems of coordinates.* Now let us look at (4), the generally covariant divergence equation. One can show that the covariant derivative takes the form

$$F_{;\nu}^{\mu\nu} = F_{,\nu}^{\mu\nu} + \Gamma_{\alpha\nu}^{\mu} F^{\alpha\nu} + \Gamma_{\beta\nu}^{\nu} F^{\mu\beta}. \tag{5}$$

Now the second term on the right hand side of (5) vanishes, because $F^{\alpha\nu} = -F^{\nu\alpha}$ and $\Gamma_{\alpha\nu}^{\mu} = \Gamma_{\nu\alpha}^{\mu}$. While for the third term on the right hand side, one can show that $\Gamma_{\beta\nu}^{\nu} = \frac{\partial \ln\sqrt{-g}}{\partial x^{\beta}}$, where g is the determinant of the metric tensor. Hence upon multiplication by $F^{\mu\beta}$ and then replacing β by ν, since it is a "dummy" index, and then combining it with the first term on the right hand side, and substituting in (4), one has

$$\frac{1}{\sqrt{-g}} \frac{\partial \sqrt{-g}\, F^{\mu\nu}}{\partial x^{\nu}} = j^{\mu}. \tag{6}$$

Now it turns out for the A.L.T. and for the L.T., as I point out in the thesis, these transformations are "unimodular" so that their determinant is -1, and hence $\sqrt{-g} = 1$. Thus the second of the two equations labelled (8.1) in the thesis holds not only for the L.T., but for the A.L.T. as well, as given in the second of the two equations labelled (8.3). Also, importantly, because of the asymmetry of the $F^{\mu\nu}$, one readily derives the continuity equation for the current four-vector

$$\frac{\partial^2 F^{\mu\nu}}{\partial x^{\mu} \partial x^{\nu}} = F_{,\mu,\nu}^{\mu\nu} = j_{,\mu}^{\mu} = 0. \tag{7}$$

What about the vector potential A_{μ}? In generally covariant form $F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu}$, and the generally covariant derivatives of the vectors are given by $A_{\mu;\nu} = A_{\mu,\nu} - \Gamma_{\mu\nu}^{\lambda} A_{\lambda}$ and $A_{\nu;\mu} = A_{\nu,\mu} - \Gamma_{\nu\mu}^{\lambda} A_{\lambda}$, and since $\Gamma_{\mu\nu}^{\lambda} = \Gamma_{\nu\,mu}^{\lambda}$, one has that the difference of the generally covariant derivatives reduces to the difference of the partial derivatives, and hence for the A.L.T. as for the L.T., or indeed for all coordinate systems in general, one has remarkably

$$F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}. \tag{8}$$

And if one substitutes this expression for the $F^{\mu\nu}$ into the first of the two equations in (8.1) of the thesis, and changes the indices in a cyclical fashion, one readily finds that the six resulting terms cancel one another, so that the equation is satisfied. Hence, as pointed out in

thesis, if we assume the two equations in (8.1) to hold, say in the rest frame, and then transform them by the A.L.T. to the moving frame, then one has exactly the same form of the equations, but with a prime on the variables, as in (8.3).

On the other hand, when one looks at the d'Alembertian wave equation for the vector potential, for convenience in the contravariant form, A^μ, the difference between the A.L.T. and the L.T. manifests itself. To obtain (8.4), we use $F^{\mu\nu} = g^{\mu\nu}g^{\nu\lambda}F_{\alpha\lambda} = g^{\mu\alpha}g^{\nu\lambda}\left(A_{\alpha,\,\lambda} - A_{\lambda,\,\alpha}\right) = = g^{\nu\lambda}A^\mu_{,\,\lambda} - g^{\mu\alpha}A^\nu_{,\,\alpha}$. At this point it is of interest to make a brief digression into general relativity. You will notice that I raised the indices on the vector potential in the last two expressions. I could do this because the metric tensor for the A.L.T. as well as the L.T. are constants, and hence their partial derivatives vanish. I could have also done this if we were working with general coordinates for which the components of the metric tensor are not constants, provided the "comma" derivative was replaced by the covariant derivative, i.e., the semicolon derivative, since the covariant derivative of the metric tensor always vanishes. But returning to flat spacetime, and the above results, when one takes the divergence of $F^{\mu\nu}$, one gets two terms: the first term is the d'Alembertian term given in (8.4) of the thesis, and the second term is $-g^{\mu\alpha}\frac{\partial}{\partial x^\alpha}\left(\frac{\partial A^\nu}{\partial x^\nu}\right)$. Then (8.4) follows if one sets $\frac{\partial A^\nu}{\partial x^\nu} = 0$; this relation is sometimes called the Landau gauge. Incidentally, you might wonder whether the Landau gauge is invariant under transforming say from the Lorentz frame to the A.L.T. frame. It turns out the gauge is invariant under all linear transformations. Proof: Let the linear coordinate transformation be given as $dx'^\lambda = a^\lambda_\nu\, dx^\nu$, and hence $A^\nu = \bar{a}^\nu_\mu A'^\mu$ with $\bar{a}^\nu_\mu a^\lambda_\nu = \delta^\lambda_\mu$. So the matrices are inverse to each other. Then $\frac{\partial A^\nu}{\partial x^\nu} = \bar{a}^\nu_\mu \frac{\partial A'^\mu}{\partial x'^\lambda}\frac{\partial x'^\lambda}{\partial x^\nu} = \bar{a}^\nu_\mu a^\lambda_\nu \frac{\partial A'^\mu}{\partial x'^\lambda} = \frac{\partial A'^\mu}{\partial x'^\mu}$. Hence, if it is the case that $\frac{\partial A^\nu}{\partial x^\nu}$ (which can also be written using the comma notation as $A^\nu_{,\,\nu}$) is chosen to vanish in one inertial frame, say the ether frame, it vanishes in any other frame connected to it by a linear coordinate transformation. Incidentally you might wonder what the situation is when we work with $A^\nu_{;\,\nu}$, that is, when we work with the covariant (or semicolon) derivative rather than just the partial (or comma) derivative? Well, from what I have written above one has $A^\nu_{;\,\nu} = A^\nu_{,\,\nu} + + \Gamma^\nu_{\alpha\nu}A^\alpha = A^\nu_{,\,\nu} + \frac{\partial \ln\sqrt{-g}}{\partial x^\alpha}A^\alpha$, and upon making use of the fact that α is a dummy index, and can be replaced by ν, the second term upon combining with the comma derivative leads to the following expression for the covariant divergence

$$A^\nu_{;\,\nu} = \frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}\,A^\nu}{\partial x^\nu} = A^\nu_{,\,\nu}\,, \tag{9}$$

for $\sqrt{-g} = 1$ or, more generally, when $\sqrt{-g}$ is a constant.

Now let us suppose we have transformed from the rest frame to the A.L.T. frame which, as above, will continue to be denoted by primes on the coordinates and field quantities, then we arrive at (8.4) in the thesis. However, the contravariant components of the metric tensor, i.e., $g'^{\mu\nu}$ were not given explicitly in (8.4). They are the inverse to the components of $g'^{\mu\nu}$ given in (1.11) of the thesis. One finds with $c = 1$ that the non-vanishing components are the following: $g'^{00} = (1 - v^2)$, $g'^{01} = g'^{10} = -v$, $g'^{11} = g'^{22} = g'^{33} = -1$. So (8.4) becomes

$$\left[(1 - v^2)\frac{\partial^2}{(\partial x'^0)^2} - 2v\frac{\partial^2}{\partial x'^0 \partial x'^1} - \frac{\partial^2}{(\partial x'^2)^2} - \frac{\partial^2}{(\partial x'^3)^2} \right] A'^{\mu} = j'^{\mu}. \quad (10)$$

This is of course different than the d'Alembertian equation in the corresponding Lorentz transformed frame, which is exactly of the same form as the rest frame. The reason for the difference is that in the A.L.T. frame, the speed of light is not the same in all directions, since the clocks have been synchronized externally so as to keep simultaneity invariant. On the other hand, as pointed out on numerous occasions before, this does not contradict the fact that the out-and-back speed is the same as for the Lorentz observer.

Since the subsequent material in the thesis through (8.7) is self-explanatory, let me go now to the equations given in (8.8). You will note that I have lowered the indices to obtain A'_0, j'_0 in terms of their contravariant expressions. Here we see another difference with the L.T., because $g_{L00} = 1$, $g_{L0i} = 0$, one has $A_{L0} = A^{L0}$, $j_{L0} = j^{L0}$ in contrast to the relations in (8.8). What is now of interest is that if we work with the mixed components, A'_0, A'^i, the transformation from the rest frame to the primed frame is exactly the same as would be the case for the the Lorentz contravariant components, A_L^0, A_L^i, and similarly for the currents, so that j'_0, j'^i are the same as j_L^0, j_L^i.

In the thesis, I then go on to say that $(j'^0, j'_x)\gamma$ are to be identified with the quantities (j_0, j_x) in the rest frame. It follows from the second equation of (8.7) that we have $j'^0\gamma = j^0$, but since in the rest frame $j^0 = j_0$, the results follow for the zeroth component. To show that the above is true for the x-component, we have $j'_x = g'_{x0}j'^0 + g'_{xx}j'^x$. Then substituting the values of the metric tensor from (1.11) we have $j'_x = -v j'^0 - (1 - v^2)j'^x$, and upon substituting from the first equation in (8.7) for j'^x into the above expression one obtains the following: $j'_x = -v j'^0 - (1 - v^2)(\gamma j^x - \gamma^2 v j'^0) = -\frac{1}{\gamma} j^x$, and since $j^x = -j_x$, upon multiplying both sides by γ the result follows.

Now let's work out some of the transformations leading to the rela-

tions in (8.10). As introduced above, the quantities a_μ^ρ and $\bar{a}_\rho^\mu$ are the transformation coefficients and their inverse for the A.L.T., and expressed in terms of partial derivatives, they are given by $a_\mu^\rho = \frac{\partial x'^\rho}{\partial x^\mu}$, $\bar{a}_\rho^\mu = \frac{\partial x^\mu}{\partial x'^\rho}$. Then, for example, $F_{01} = a_0^0 a_1^1 F'_{01} + a_0^1 a_1^0 F'_{10}$, but since $a_1^0 = \frac{\partial x'^0}{\partial x^1} = 0$, unlike the case for the L.T., one has $F_{01} = \gamma \frac{1}{\gamma} F'_{01} = F'_{01}$ as in the thesis, but with the subscript "1" replaced by the letter "x", and recalling $c = 1$, $x^0 = t$, $x'^0 = t'$. Let us now derive the contravariant expressions given by $F^{01} = \bar{a}_0^0 \bar{a}_1^1 F'^{01} + \bar{a}_1^0 \bar{a}_0^1 F'^{10}$. Once again, the second term on the right hand side vanishes for the A.L.T., and since its inverse is given by $x = \frac{1}{\gamma} x' + v\gamma t'$, $t = \gamma t'$, it follows that $\bar{a}_0^0 = \frac{\partial x^0}{\partial x'^0} = \gamma$ and that $\bar{a}_1^1 = \frac{1}{\gamma}$, hence it follows that $F^{01} = \gamma \frac{1}{\gamma} F'^{01} = F'^{01}$.

Thus we see that the covariant and contravariant forms of the antisymmetric electromagnetic field tensor are invariant under the A.L.T. in the direction of motion! This is also true for the L.T. as shown in the two top relations in (8.13), and as I will prove here explicitly further below. But physically, why is this the case? The argument that I have heard goes as follows. Let us imagine electric charge spread uniformly on an infinite plane metal surface, which we will take to be the yz plane. The electric field is uniform, and given by E_x, and in suitable units is just the surface charge density. Now look at the field in a frame travelling in the x-direction, *i.e.*, normal to the plane. Since the electric charge is conserved, as discussed in the paragraph following (8.4), and since the y and z coordinates are left invariant under both the L.T. and the A.L.T., then the surface charge density is invariant, and hence the electric fields are invariant, which explains physically why the two top equations for the electric fields in (8.13) come out the same for the two transformations. I have only discussed here the electric fields in the direction of motion, and you might find it interesting to work out the case for the electric fields in the y and z directions as given in (8.10).

But before going on to the transformation that links the L.T. fields with the A.L.T. fields, let us look at the transformation for the covariant and contravariant components of the magnetic fields as given in (8.11). Now F_{yz} is the magnetic field in the x-direction in the rest frame, and we see it is the same as in the A.L.T. frame. Mathematically, this comes about because $F_{yz} = a_y^\mu a_z^\nu F'_{\mu\nu}$, and since $a_y^\mu = \delta_y^\mu$, $a_z^\nu = \delta_z^\nu$ from the A.L.T., the result given in the thesis follows. A similar argument holds for F^{yz}. Physically, this means the magnetic field in the direction of motion is invariant under the A.L.T. as it is under the L.T. One can show that this should be the case by an argument similar to the argument for the electric field by thinking in terms of little current loops

lying in the yz plane, and using the fact that the y and z coordinates are not changed under either transformation. Once again you may wish to work out the case for the other components of the magnetic field as given in the remainder of the relations in (8.11).

Now let us turn to comparing the fields under the A.L.T. with those under the L.T., and in order to be clear as to the physical circumstance under which the comparison is being made, imagine we are on a train travelling with velocity v in the x-direction with the station taken as the rest frame. On the train there are two sets of clocks: one set has been synchronized internally, either by the Einstein method, or by slowly moving them. According to these clocks, the one-way speed of light is the same in all directions. These are the clocks that obey the Lorentz transformation. The second set of clocks are those associated with the observers using the A.L.T. who have synchronized their clocks with those in the station. As discussed in the thesis, the transformation connecting the A.L.T. with the L.T. is a local time transformation: $t_L = t' - vx'$, $x_L = x'$, $y_L = y'$, $z_L = z'$, which in differential tensorial form as given in (8.12) is written $dx_L^\mu = \ell_\nu^\mu \, dx'^\nu$, and the inverse transformation is $dx'^\mu = \bar{\ell}_\nu^\mu \, dx_L^\nu$. Let us work out explicitly $F_{L01} = \bar{\ell}_0^\mu \bar{\ell}_1^\nu F'_{\mu\nu}$. Now rewrite the local time transformation as $t' = t_L + vx_L$, $x' = x_L$, $y' = y_L$, $z' = z_L$, so that $\bar{\ell}_0^\mu = \frac{\partial x'^\mu}{\partial x_L^0} = \delta_0^\mu$, while $\bar{\ell}_1^\nu = \frac{\partial x'^\nu}{\partial x_L^1}$ has two nonzero values given by $\bar{\ell}_1^0 = v$, and also $\bar{\ell}_1^1 = 1$. However, because $F'_{\mu\nu}$ is antisymmetric, $F'_{00} = 0$, and therefore the only term that survives corresponds to $F_{L01} = \bar{\ell}_0^0 \bar{\ell}_1^1 F'_{01} = F'_{01}$ as given in the top left relation in (8.13). One can of use the same analysis based on the local time transformation to derive the rest of the relations I have given there.

What is very interesting is that we see that the covariant form of the electric field (*i.e.*, with both indices lowered as given in the upper left column of (8.13) for the L.T. is exactly the same as for the corresponding electric field for the A.L.T. This can be summarized in the following way: $F_{L0i} = F'_{0i}$, $i = 1, 2, 3$, or, $i = x, y, z$, as in the thesis. On the other hand, when it comes to the magnetic field, as is clear from the lower right hand column in (8.14), it is the contravariant components describing the magnetic field that are the same for both transformations. It follows that when we use the covariant components of the e-m field tensor for the electric field, and the contravariant components for the magnetic field, *the transformation from the rest frame to the primed frame is exactly the same as for the Lorentz transformation.* We have already shown this is the case for the components in the direction of motion, but now let us look at the transverse components, and specifically, the y-component,

since by isotropy, the result will hold for the z-component as well.

So let us go back to (8.10) and look at

$$F_{0y} = \frac{1}{\gamma} F'_{0y} - v\gamma F'_{xy}. \tag{11}$$

We want to rewrite this so that instead of F'_{xy} being present in (11), we have F'^{xy}, and then verify that this relation has the same form as for the Lorentz transformation. We use $F'_{xy} = g'_{x\mu} g'_{y\nu} F'^{\mu\nu} = g'_{x0} g'_{yy} F'^{0y} + + g'_{xx} g'_{yy} F'^{xy}$, since all other terms vanish. Note that I have use x, y instead of 1, 2 as indices at this point so as to make it easier to compare with the thesis. Next, substituting values from the A.L.T. metric given in (1.11) one finds

$$F'_{xy} = v F'^{0y} + (1 - v^2) F'^{xy}. \tag{12}$$

However we see that we have now introduced F'^{0y} which we do not want. So we now use the following relation $F'^{0y} = g'^{0\mu} g'^{y\nu} F'_{\mu\nu} = = g'^{00} g'^{yy} F'_{0y} + g'^{0x} g'^{yy} F'_{xy} = -(1 - v^2) F'_{0y} + v F'_{xy}$, which we now substitute in (12) to obtain

$$F'_{xy} = -v(1 - v^2) F'_{0y} + v^2 F'_{xy} + (1 - v^2) F'^{xy}. \tag{13}$$

Upon bringing the term $v^2 F'_{xy}$ over to the left hand side and solving, one finds that F'_{xy} can be written as

$$F'_{xy} = -v F'_{0y} + F'^{xy}. \tag{14}$$

Now substitute (14) in (11), so that we have $F_{0y} = \frac{1}{\gamma} F'_{0y} - v\gamma \times \times (-v F'_{0y} + F'^{xy})$, and rearranging terms, we have $F_{0y} = \gamma \left(\frac{1}{\gamma^2} + v^2\right) F'_{0y} - - v\gamma F'^{xy}$, and using $\frac{1}{\gamma^2} + v^2 = 1$, we finally have that

$$F_{0y} = \gamma \left(F'_{0y} - v F'^{xy}\right), \tag{15}$$

which is exactly the transformation for the corresponding L.T. quantities, *i.e.*, one has that

$$F_{0y} = \gamma \left(F_{L0y} - v F_L^{xy}\right), \tag{16}$$

which can be obtained directly by employing the L.T. for the covariant components, and noting that for the L.T., unlike the case for the A.L.T., one has

$$F_{Lxy} = g_{Lx\mu} g_{Ly\nu} F_L^{\mu\nu} = (-1)(-1) F_L^{xy} = F_L^{xy}, \tag{17}$$

since all the off-diagonal terms vanish, and the diagonal terms for the spatial components are equal to -1. And as we have shown, using the

local time transformation, $F'_{0y} = F_{L0y}$, and one can show in the same way, $F'^{xy} = F^{xy}_L$, and hence the equivalence of the results for the A.L.T. and the L.T. is established.

It is interesting to note from (15) and (16) that while F'_{0y}, F'^{xy} are equivalent to what the L.T. observer says are the electric and magnetic fields in the x and z directions, respectively, the A.L.T. observer says that in addition, there is another projection of the magnetic field that involves the velocity relative to the rest frame as well as the electric field. Thus from (8.14) one has $F_{Lxy} = F'_{xy} + v F'_{0y}$, and from (8.13) one has $F'_{0y} = F_{L0y}$, so that finally one has

$$F'_{xy} = F_{Lxy} - v F_{L0y} \,. \tag{18}$$

However, unless one has some way of setting up an external synchronization with the rest frame, F'_{xy} is strictly *unobservable*. This is the same situation that exists for the one-way velocity of light: if one has no way of making an external synchronization, one relies on either Einstein synchronization, or that with slowly moved clocks, in which case the speed of light is c in all directions.

I think the remainder of section (8.1) in the thesis is self-explanatory, but here is an additional comment that has bearing on section (8.2) that deals with the equations of motion of a charged particle. Let us suppose in a frame travelling uniformly with speed v in the x-direction relative to the rest frame, one does an experiment, say with electrons in an electromagnetic field, causing them to travel along some path, chosen for simplicity to be in the plane $z_L = z' = 0$. Let us suppose the L.T. observer finds an electron follows a path given by $y_L = f(x_L)$. Then under the local time transformation connecting the A.L.T. with the L.T., we must have $y' = f(x')$, so the electron will travel on the same path for the A.L.T. observer as for the L.T. observer. However, if the electron left the point A at the L.T. time $t_L(A)$, and arrived at the point B at the time $t_L(B)$, these times will in general be *different* for the A.L.T. observer, who will assign them times corresponding to the local time transformation. For convenience, one can assume initially that the A.L.T. observer's clock at A has been seen set to agree with that of the L.T. observer's clock, so that $t'(A) = t_L(A)$. Then, assuming the electron does not travel on a closed path, one has $t'(B) = t_L(B) + v x_L(B)$, so that the two observers assign different travel times to the electron for the *same* path, just as the A.L.T. observer assigns a different travel time for light than the L.T. for the same path, provided it is not closed. For a closed path, they of course agree, since the synchronization of separated clocks is not involved, and the A.L.T. clock and the L.T. clock both

keep time at the same rate. This underlies the agreement between the A.L.T. and the L.T. for the Michelson-Morley and Kennedy-Thorndike experiments.

Finally, I would like to remark, as I noted in my 1961 *Supplemento al Nuovo Cimento* article, that the A.L.T. is useful in teaching students about the meaning of general covariance in the simple case when the metric tensor is not diagonal, but its coefficients are all constants, so that all the Christoffel symbols vanish, and the covariant derivatives reduce to ordinary partial derivatives. Also, there is an analogy with quantum mechanics, in that one sees that the A.L.T., because of the different synchronization of clocks from the L.T., splits the degeneracy between covariant and contravariant components which, under the L.T., apart from a possible minus sign, are the same. So in the case of the A.L.T., the student has to confront the different physical interpretation of these now degeneracy-split terms, that would not be the case if one only dealt with the L.T., and this can help to stimulate new insights into special relativity and electromagnetic theory, and possibly even suggest new experiments to be performed.

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