The Generalized Warp Drive Concept in the EGR Theory

Patrick Marquet*

Abstract: In this paper, we briefly review the basic theory of the Alcubierre drive, known as the Warp Drive Concept, and its subsequent improvements. By using the Arnowitt-Deser-Misner formalism we then re-formulate an extended extrinsic curvature which corresponds to the extra curvature of the Extended General Relativity (EGR). With this preparation, we are able to generalize the Alcubierre metric wherein the space-like hypersurfaces are Riemannian, and the characteristic Alcubierre function is associated with the EGR geometry. This results in a reduced energy density tensor, whose form displays a potential ability to avoid the weak energy condition.

Contents:

*Postal address: 7, rue du 11 nov, 94350 Villiers/Marne, Paris, France. E-mail: patrick.marquet6@wanadoo.fr. Tel: (33) 1-49-30-33-42.

Notations:

To completely appreciate this article, it is imperative to define some notations employed.

Indices. Throughout this paper, we adopt the Einstein summation convention whereby a repeated index implies summation over all values of this index:

4-tensor or 4-vector: small Latin indices $a, b, \ldots = 1, 2, 3, 4$;

3-tensor or 3-vector: small Greek indices $\alpha, \beta, \ldots = 1, 2, 3$;

4-volume element: d^4x;

3-volume element: d^3x.

Signature of space-time metric:

$(-+++)$ unless otherwise specified.

Operations:

Scalar function: $U(x^a)$;

Ordinary derivative: $\partial_a U$;

Covariant derivative in GR: ∇_a;

Covariant derivative in EGR: D_a or $'$, (alternatively).

Newton's constant:

$\mathfrak{G} = c = 1$.

Introduction

The physical restriction related to the finite nature of the light velocity has so far been a stumbling block to exploring the superluminal speed possibility of long-term space journeys.

However, recent theoretical works have lent support to plausible interstellar hyperfast travels, without physiological human constraints.

How is this possible? The principle of space travel while locally "at rest", is analogous to galaxies receding away from each other at extreme velocities due to the expansion (and contraction) of the Universe.

Instead of moving a spaceship from a planet A to a planet B, we modify the space between them. The spaceship can be carried along by a local spacetime "singular region" and is thus "surfing" through space with a given velocity with respect to the rest of the Universe.

In 1994, a Mexican physicist Miguel Alcubierre [1], working at the Physics and Astronomy Department of Cardiff University in Wales, Great Britain, published a short paper describing such a propulsion mode, known today under the name *Warp Drive*.

Based on this theory, a faster than light travel could be for the first time considered without violating the laws of relativity.

Many problems (open questions) remain to be investigated, among which two major problems are reflected in the following statements:

a) Produce a sufficiently large negative energy to create a local space distortion without violating the energy conditions resulting from the laws of General Relativity [2];
b) Maintain contact (control) between the spaceship and the outside of the distorsion (causality connection).

The problem a) can be avoided if one considers a non-Riemannian geometry that governs the laws of our Universe [3] which could eliminate the negative energy density required by the Alcubierre metric to sustain a realistic Warp Drive.

The difficulty b) may be theoretically circumvented by introducing certain types of transformations which may allow us to use the warped regions for the removal of the singularities or "event horizons". Some of these transformations are briefly reviewed in the course of this study.

Pre-requisite: time-like unit four-vector

As is well known [4], the covariant derivative of a time-like vector field u^a (whose square is $u^a u_a = -1$), may be expressed in an invariant manner in terms of tensor fields which describe the kinematics of the congruence of curves generated by the vector field u^a.

One may write

$$u_{a;b} = \varsigma_{ab} + \omega_{ab} + \frac{1}{3}\left(\theta h_{ab}\right) + \dot{u}_a u_b\,,$$

where $\dot{u}_a = u_{a;b}u^b$ is the acceleration of the flow lines, τ is the proper time, $\omega_{ab} = h^c_a h^d_b u_{[c;d]}$ is the vorticity tensor, $h_{ab} = g_{ab} + u_a u_b$ is the projection tensor, $\theta_{ab} = h^c_a h^d_b u_{(c;d)}$ is the expansion tensor, $\theta = h^{ab}\theta_{ab} = u^a_{;a}$ is the expansion scalar, $\varsigma_{ab} = \theta_{ab} - \frac{1}{3}\left(h_{ab}\theta\right)$ is the shear tensor.

The kinematic quantities are completely orthogonal to u^a, i.e.,

$$h_{ab}u^b = \omega_{ab}u^b = \varsigma_{ab}u^b = 0\,, \quad \dot{u}_a u^b = \omega_a u^b = 0\,.$$

Physically, the time-like vector field u^a is often taken to be the four-velocity of a fluid. The volume element expansion θ extracted from this decomposition can be thus seen as a hydrodynamic picture: it is of major importance in the foregoing.

Chapter 1. Basics of Warp Drive Physics

§1.1. Description of the Alcubierre concept

§1.1.1 Space-time bubble

The Universe is approximated as a Minkowskian space: we choose an arbitrary curve and deform the space-time in the immediate vicinity in such a way that the curve becomes a time-like geodesic somewhat like a "ripple", in order to generate a perturbed or singular local region in which one may fit a spaceship and its occupants.

Let x_s be the center of the region where the spaceship stays, and x any coordinate within this region so that $x = x_s$ for the spaceship.

Within an orthonormal coordinate frame, such a region, which is referred to as a *bubble*, is transported forward with respect to distant observers, along a given direction (x in this text).

With respect to the same distant observers, the apparent velocity of the bubble center is given by

$$v_s(t) = \frac{dx_s(t)}{dt}\,, \tag{1.1}$$

where $x_s(t)$ is the trajectory of the region along the x-direction, and

$$r_s(t) = \sqrt{\left(x - x_s(t)\right)^2 + y^2 + z^2} \tag{1.2}$$

is the variable distance outward from the center of the spaceship until $\Re$ which may be called the *radius of the singular region.*

The spaceship is at rest inside the bubble and has no local velocity.

§1.1.2. Characteristics

From these first elements, we must now select the exact form of a metric that will "push" the spaceship along a trajectory described by an arbitrary function of time (x_s, t).

Furthermore, this trajectory should be a time-like geodesic, whatever $v_s(t)$. By substituting $x = x_s(t)$ in the new metric to be defined, we should expect to find

$$d\tau = dt\,. \tag{1.3}$$

The proper time of the spaceship is equal to coordinate time which is also the proper time of distant observers.

Since these observers are situated in the flat region, we conclude that the spaceship suffers no time dilation as it moves. It will be easy to prove that this spaceship moves along a time-like geodesic and its proper acceleration is zero.

§1.2. The physics that leads to Warp Drive

§1.2.1. The (3+1) Formalism: the Arnowitt-Deser-Misner (ADM) technique

In 1960, Arnowitt, Deser, and Misner [5] suggested a technique based on decomposing the space-time into a family of space-like hypersurfaces and parametrized by the value of an arbitrarily chosen time coordinate x^4.

This "foilation" displays a proper-time element $d\tau$ between two nearby hypersurfaces labelled $x^4 = const$ and $x^4 + dx^4 = const$. The proper-time element $d\tau$ must be proportional to dx^4. Thus we write

$$d\tau = N\big(x^\alpha, x^4\big)\, dx^4. \tag{1.4}$$

In the ADM terminology, N is called the *lapse function* and more specifically the *time lapse*.

Consider now the three-vector whose spatial coordinates x^α are lying in the hypersurface ($x^4 = const$) and which is normal to it.

We want to evaluate this vector on the second hypersurface, which is $x^4 + dx^4 = const$, where these coordinates now become $N^\alpha dx^4$. This N^α vector is known as the *shift vector*.

The ADM four-metric tensor is decomposed into covariant components

$$(g_{ab})_{\text{ADM}} = \begin{cases} -N^2 - N_\alpha N_\beta\, g^{\alpha\beta}, & N_\beta\,, \\ \qquad N_\alpha\,, & g_{\alpha\beta}\,. \end{cases} \tag{1.5}$$

The line element corresponding to the hypersurfaces' separation is therefore written

$$(ds^2)_{\rm ADM} = (g_{ab})_{\rm ADM}\, dx^a dx^b$$

or

$$(ds^2)_{\rm ADM} = -N^2 \left(dx^4\right)^2 + g_{ab}\left(N^\alpha dx^4 + dx^\alpha\right)\left(N^\beta dx^4 + dx^\beta\right) = \\ = \left(-N^2 - N_\alpha N^\alpha\right)\left(dx^4\right)^2 + 2N_\beta\, dx^4 dx^\beta + g_{\alpha\beta}\, dx^\alpha dx^\beta, \quad (1.6)$$

where $g_{\alpha\beta}$ is the 3-metric tensor of the hypersurfaces.

The ADM metric tensor has the contravariant components

$$(g^{ab})_{\rm ADM} = \begin{cases} -N^{-2}, & \dfrac{N^\beta}{N^2}\,, \\ \dfrac{N^\alpha}{N^2}\,, & g^{\alpha\beta} - N^\alpha \dfrac{N^\beta}{N^2}\,. \end{cases} \quad (1.7)$$

As a result, the hypersurfaces have a unit time-like normal vector with components

$$n^a = N^{-1}\left(1,\, -N^\alpha\right), \qquad n_a = \left(-N,\, 0\right). \quad (1.8)$$

When the fundamental three-tensor satisfies $g^{\alpha\beta} = \delta^{\alpha\beta}$ the metric (1.6) becomes

$$ds^2 = -\left(N^2 - N_\alpha N^\alpha\right)dt^2 - 2N^\alpha dx\, dt + dx^\alpha dx^\beta$$

or

$$ds^2 = -N^2 dt^2 - \left(dx + N^\alpha dt\right)^2 + dy^2 + dz^2. \quad (1.9)$$

§1.2.2. Curvatures in the ADM formalism

The Einstein action can be written in terms of the metric tensor $(g_{ab})_{\rm ADM}$ (1.5) and (1.7), as [6]

$$S_{\rm ADM} = \int R\sqrt{-g}\; d^4x = \\ = \int dt \int N\left(K_{\alpha\beta}K^{\alpha\beta} - K^2 + {}^{(3)}\!R\right)\sqrt{-g}\; d^3x + \\ + \text{boundary terms}\left(K^\alpha_\alpha K^\beta_\beta = K^2\right), \quad (1.10)$$

where $g = \det \| g_{\alpha\beta} \|$, while ${}^{(3)}\!R$ stands for the *intrinsic curvature* tensor of the hypersurface $x^4 = const$

$$K_{\alpha\beta} = (2N)^{-1}\left(-N_{\alpha;\beta} - N_{\beta;\alpha} + \partial_t\, g_{\alpha\beta}\right). \quad (1.11)$$

The tensor (1.11) (in which ; refers to covariant differentiation with respect to the three-metric), represents the *extrinsic curvature*, and as such, describes the manner in which that surface is embedded in the surrounding four-dimensional space-time.

The determinant ${}^{(4)}g$ of the four-metric is shown to be related to the determinant ${}^{(3)}g$ by

$$\sqrt{-\,{}^{(4)}g} = N\,\sqrt{{}^{(3)}g}\ .$$

The rate of change of the three-metric tensor $g_{\alpha\beta}$ with respect to the time label can be decomposed into "normal" and "tangential" contributions:

- The normal change is proportional to the extrinsic curvature $\frac{-2N}{K_{\alpha\beta}}$ of the hypersurface;
- The tangential change is given by the Lie derivative of $g_{\alpha\beta}$ along the shift vector N^{α}, namely

$$\underset{N}{\mathrm{L}}\, g_{\alpha\beta} = 2N_{(\alpha;\beta)}\,. \tag{1.12}$$

The main advantage of the ADM formalism is that the time derivative is isolated and it can be used in further specific computations. Furthermore we verify that

$$K_{\alpha\beta} = -\,n_{\alpha;\beta}\,, \tag{1.13}$$

which is sometimes called the *second fundamental form* of the three-space [7]. Six of the ten Einstein equations imply for K^{α}_{β} to evolve according to [8]

$$\frac{\partial K^{\alpha}_{\beta}}{\partial t} + \underset{N}{\mathrm{L}}\, K^{\alpha}_{\beta} = \nabla^{\alpha}\nabla_{\beta} N + \\ + N\left[R^{\alpha}_{\beta} + K^{\alpha}_{\alpha}K^{\alpha}_{\beta} + 4\pi\left(T - C\right)\delta^{\alpha}_{\beta} - 8\pi T^{\alpha}_{\beta}\right], \tag{1.14}$$

where R^{α}_{β} is the three-Ricci tensor, and $C = T_{ab}\, n^{a} n^{b}$ is the material energy density in the rest frame of normal congruence (time-like vector field) with $T = T^{\alpha}_{\alpha}$.

It is convenient to introduce the three-momentum current density $I_{\alpha} = -\,n_{c}\, T^{c}_{\alpha}$. So the remaining four equations finally form the so-called *constraint equations*

$$H = \frac{1}{2}\left(R - K^{\alpha}_{\beta}K^{\beta}_{\alpha} + K^{2}\right) - 8\pi C = 0\,, \tag{1.15}$$

$$H_{\beta} = \nabla_{\alpha}\left(K^{\alpha}_{\beta} - K\delta^{\alpha}_{\beta}\right) - 8\pi I_{\beta} = 0\,. \tag{1.16}$$

Equation (1.15) will be of central importance in the present theory.

Chapter 2. The Alcubierre Warp Drive

§2.1. The Alcubierre metric

In view of building a space warp progressing along the x-direction, one may choose with Alcubierre

$$\left.\begin{aligned} &N = 1 \\ &N^1 = -\, v_s(t)\, f(r_s,t) \\ &N^2 = N^3 = 0 \end{aligned}\right\}, \tag{2.1}$$

we then have

$$(ds^2)_{\text{AL}} = -\, dt^2 + \big[dx - v_s\, f(r_s,t)\, dt\big]^2 + dy^2 + dz^2; \tag{2.2}$$

this interval is known as the *Alcubierre metric*.

The function $f(r_s,t)$ is so defined as to cause space-time to contract on the forward edge and equally expanding on the trailing edge of the singular region. It is often referred to as a "*top hat*" *function*.

Let us now write down the Alcubierre metric under the following equivalent form

$$(ds^2)_{\text{AL}} = -\,\big[1 - v_s^2 f^2(r_s,t)\big]\, dt^2 - 2\, v_s f\, dt\, dx + dx^2 + dy^2 + dz^2, \tag{2.3}$$

which puts in evidence the covariant components of the Alcubierre metric tensor

$$\left.\begin{aligned} &(g_{44})_{\text{AL}} = -\,\big[1 - v_s^2 f^2(r_s,t)\big] \\ &(g_{41})_{\text{AL}} = (g_{14})_{\text{AL}} = -\, v_s\, f(r_s,t) \\ &(g_{22})_{\text{AL}} = (g_{33})_{\text{AL}} = 1 \end{aligned}\right\}. \tag{2.4}$$

§2.2 Analyzing the "top hat" function

We now turn our attention to the "top hat" function $f(r_s,t)$ itself, which allows for the bubble to develop. Alcubierre originally chosen the following form

$$f(r_s,t) = \frac{\tanh\big[\sigma\,(r_s + \Re)\big] - \tanh\big[\sigma\,(r_s - \Re)\big]}{2\tanh\,(\sigma R)}\,, \tag{2.5}$$

where $\Re > 0$ is the "radius" of the "region", while σ is a "bump" parameter which can be used to "tune" the "wall" thickness of the singular region.

The larger this parameter, the greater the contained energy density, so its shell thickness decreases. Moreover, the absolute increase of σ means a faster approach of the condition

$$\lim_{\sigma\to\infty} f(r_s,t) = \begin{cases} 1 \ \text{ for } \ r_s \in [-\Re,\, \Re\,], \\ 0 \text{ otherwise.} \end{cases} \tag{2.6}$$

Note that $r_s = 0$ at the center of the singular region (spaceship location). For $r_s > \Re$, the function $f(r_s,t)$ should rapidly verify $f(r_s,t) = 0$ and we recover the Minkowski space-time.

As outlined earlier, any function will suffice so long as the above conditions are fullfilled. For simplified calculations, it is convenient to introduce the equivalent *piecewise continuous function* as established by Pfenning and Ford [9]

$$f_{\text{p.c.}}(r_s,t) = \begin{cases} 1 \ \text{ for } \ r_s < \Re - \frac{\Delta}{2}\,, \\ \left(\frac{-1}{\Delta}\right)\left(r_s - \Re - \frac{\Delta}{2}\right) \ \text{ for } \ \Re - \frac{\Delta}{2} < r_s < \Re + \frac{\Delta}{2}\,, \\ 0 \ \text{ for } \ r_s > \Re + \frac{\Delta}{2}\,, \end{cases} \tag{2.7}$$

where the variable Δ is the region shell "thickness".

Setting the slopes of the functions $f(r_s,t)$ and $f_{\text{p.c.}}(r_s,t)$ to be equal at $r_s = \Re$, leads to the following result

$$\Delta = \frac{1 + \tanh^2(\sigma\Re)^2}{2\sigma\,[\,\tanh(\sigma\Re)]}\,. \tag{2.8}$$

For large $\sigma\Re$, one may admit the approximation

$$\Delta \approx \frac{2}{\sigma}\,. \tag{2.9}$$

§2.3. Eulerian observer

§2.3.1 Definition

With the choice of the three-vector $N^\alpha = 0$, we have a particular coordinate frame called *normal coordinates*, according to (1.8). Such a choice of coordinates constitutes an "Eulerian" gauge.

In the Alcubierre formalism, $N^1 \neq 0$ characterizes a special type of observer who "measures" the warped shell and the associated region when they cross through.

His four-velocity is normal to the hypersurfaces. This observer, who also is referred to as *Eulerian observer*, is initially at rest. Just the front wall of the disturbance reaches the observer, he begins to accelerate, in the progressing direction of the singular region, relative to observers located at large distance from him.

Once during his "stay" inside the region, the Eulerian observer travels with a nearly constant velocity given by

$$\frac{dx(t)}{dt} = v_s(t_\rho, \rho)\, f(\rho)\,, \tag{2.10}$$

where t_ρ is the time measured at the coordinate

$$\rho = \sqrt{y^2 + z^2}\ . \tag{2.11}$$

This velocity will always be less than the region's velocity unless $\rho = 0$, i.e. when the observer is at the center of the spaceship.

After reaching the region's equator, the Eulerian observer decelerates, and is left at rest while going out of the rear edge of the "wall".

If using the piecewise continuous function of Pfenning for $r_s < \Re - \frac{\Delta}{2}$, any observer moves along the singular region with the same speed. Inside the warped regions ("shells"), i.e. for

$$\Re - \frac{\Delta}{2}\ <\ \rho\ <\ \Re + \frac{\Delta}{2}\,,$$

we recover the conditions deduced from the "top hat" function (2.5), as viewed by the Eulerian observer. The singular regions have toroidal geometry concentrated on either part of the longitudinal direction of travel x, and are thus perpendicular to the plane defined by ρ.

§2.3.2. Specific characteristics

Following Alcubierre, such an observer has a four-velocity normal to the hypersurfaces $t = const$.

With the condition $d\tau = dt = ds$, it is straightforward to show that this four-velocity has the following components

$$\left.\begin{aligned} (u^a)_{\rm AL} &= \big[\,1,\, v_s\, f(r_s, t),\, 0,\, 0\,\big] \\ (u_a)_{\rm AL} &= \big[\,-1,\, 0,\, 0,\, 0\,\big] \end{aligned}\right\}. \tag{2.12}$$

The Eulerian observer follows time-like geodesics orthogonal to the Euclidean hypersurfaces.

From the metric (2.2), inspection shows that the Eulerian observer is in free fall, i.e. his four-acceleration is zero

$$(a^b)_{\rm AL} = (u^a)_{\rm AL}\,(u^b_{;a})_{\rm AL} = 0\,,$$

which confirms the postulate of §1.1.2.

In this case $\delta^{\alpha\beta} = g^{\alpha\beta}$, $N = 1$, and (1.11) reduces to

$$K_{\alpha\beta} = \frac{1}{2}\left(\partial_\alpha N_\beta + \partial_\beta N_\alpha\right).$$

The contracted tensor, which is defined by

$$\theta = -\operatorname{trace} K^{\alpha\beta}, \tag{2.13}$$

is the *expansion scalar* defined above; it means the expansion of the three-volume element which, taking account of (2.1), is

$$\theta = v_s\,\frac{df}{(dx)_{\mathrm{AL}}}\,, \tag{2.14}$$

where $(x)_{\mathrm{AL}} = x - x_s(t)$ is the single derivative variable.

Hence, we find

$$\theta = v_s\left(\frac{df}{dr_s}\right)\left[\frac{dr_s}{d(x-x_s)}\right] \tag{2.15}$$

and by using the classical derivative formula of functions of functions, it is not difficult to show that this last formula becomes

$$\theta = v_s\left(\frac{df}{dr_s}\right)\left(\frac{x_s}{r_s}\right). \tag{2.16}$$

Obviously, the shape of the function f, (2.5) induces both a volume contraction and expansion ahead of, as well as behind, the singular region.

§2.4. Negative energy requirement

§2.4.1. The Alcubierre-Einstein tensor

Before determining the form of the Alcubierre-Einstein tensor, we recall briefly the so-called *energy conditions.*

Let us consider at a point p on the manifold (M, g_{ab}), an energy-momentum tensor T^{ab}.

For any time-like vector $u^a \in \mathrm{T}_p$ (tangent space at p), one must have the inequality

$$C = T_{ab}\,u^b u^b \geqslant 0\,, \tag{2.17}$$

known as the *weak energy condition.*

In addition, the "*dominant*" *energy condition* stipulates that for any time-like four-vector $u^a > 0$, the four-vector $Q^a = T^a_b u^b$ is a non-space-like vector.

By continuity, the weak energy condition implies the *null energy condition* which asserts that for any null vector k^a

$$T_{ab}\,k^a k^b \geqslant 0\,.$$

Lastly, we consider the *strong energy condition* for any time-like four-vector u^a

$$\left(T_{ab} - \frac{1}{2}\,g_{ab}\,T\right) u^a u^b \geqslant 0\,.$$

NOTE: The dominant energy condition implies the weak energy condition and therefore the null energy condition, but not necessarily the strong energy condition, which itself implies the null energy condition but not necessarily the weak energy condition.

From the components of the metric tensor (2.4), it is possible to form the contravariant components of the Ricci tensor $(R^{ab})_{\rm AL}$ of the Alcubierre metric.

The resulting Einstein tensor

$$(G^{ab})_{\rm AL} = (R^{ab})_{\rm AL} - \frac{1}{2}\,(g^{ab})_{\rm AL}\,R$$

contains the time component $(R^{44})_{\rm AL}$ and

$$(G^{44})_{\rm AL} = -\left(\frac{v_s^2}{4r_s^2}\right)\rho^2\left(\frac{df}{dr_s}\right)^2.$$

Using $(G^{44})_{\rm AL}$ to define the energy density $(T^{44})_{\rm AL}$, one finds

$$C = \frac{1}{8\pi}\,(G^{44})_{\rm AL}\,(u_4\,u_4)_{\rm AL} = -\frac{1}{32\,\pi}\left(\frac{v_s^2\rho^2}{r_s^2}\right)\left(\frac{df}{dr_s}\right)^2. \tag{2.18}$$

This formula is always negative as seen by the Eulerian observers, and therefore it is not compatible with the energy condition (2.17).

Another way of writing this equation is obtained by using the Gauss-Codazzi relations to form the Einstein tensor as a function of both the intrinsic and extrinsic curvatures, which eventually leads to [10]

$$C = T_{ab}\,n^a n^b = \frac{1}{16\pi}\left({}^{(3)}R + K^2 - K_{\alpha\beta}K^{\alpha\beta}\right). \tag{2.19}$$

By choosing $N^1 = -v_s\,f(r_s)$, $N^2 = N^3 = 0$, and ${}^{(3)}R = 0$ the Alcubierre formulation is obtained again.

The energy density as measured by the Eulerian observer is given by

$$(C)_{\rm AL} = \frac{1}{16\pi}\left(K^2 - K_{\alpha\beta}K^{\alpha\beta}\right), \tag{2.20}$$

thus referring to (2.13), we find back

$$\theta = -\,\partial_1 N^1 = v_s\, f'(r_s)\, \frac{x - x_s}{r_s} \tag{2.21}$$

and

$$(C)_{\text{AL}} = \frac{1}{16\pi}\left[\left(\partial_1 N^1\right)^2 - \left(\partial_1 N^1\right)^2 - 2\left(\frac{\partial_2 N^1}{2}\right)^2 - 2\left(\frac{\partial_3 N^1}{2}\right)^2\right], \tag{2.22}$$

$$(C)_{\text{AL}} = -\frac{1}{32\pi}\, v_s^2\, f'^2(r_s)\, \frac{y^2 + z^2}{r_s^2}\,. \tag{2.23}$$

§2.4.2. Negative energy

We now write down the form of the total negative energy required to sustain the Alcubierre metric.

Without loss of generality, we may simplify the case by assuming a constant velocity for the singular region, i.e.

$$x(t) = v_s\,(t) \tag{2.24}$$

at $t = 0$, we have

$$r_s(t = 0)\,, \qquad \sqrt{(x^\alpha)^2} = r\,. \tag{2.25}$$

Under these conditions, we must calculate the integral of the local energy density over the proper volume $d^3x = dV$ (hypersurface)

$$E = \int \sqrt{y}\; T^{44}\, dV, \tag{2.26}$$

where y is the determinant of the spatial metric on the hypersurface $t = const$, which, in our case, is $y = 1$.

One finds

$$E = -\,\frac{1}{32\pi}\, v_s^2 \int \frac{\rho^2}{r^2}\left[\frac{df(r_s,t)}{dr}\right]^2 dV. \tag{2.27}$$

With the piecewise function of Pfenning (2.7), the energy is, in the spherical coordinates

$$E = -\,\frac{1}{12}\, v_s^2 \int_{\Re-\Delta/2}^{\Re+\Delta/2} r^2 \left(-\frac{1}{\Delta}\right)^2 dr. \tag{2.28}$$

The contributions to the energy come only from the singular region's "shell" areas.

We then see that one needs a special type of negative energy (matter) to travel faster than the speed of light by means of a Warp Drive. Such an exotic matter has never been detected so far.

Chapter 3. Causality

§3.1. Horizon formation

We regard the speed of the spaceship v as constant, and r_s is then

$$r_s = \sqrt{(x - vt)^2 + y^2 + z^2} \tag{3.1}$$

reducing the metric (2.3) to two dimensions, $y = z = 0$, we obtain

$$ds^2 = -\left(1 - v^2 f^2\right) dt^2 - 2vf\,dx\,dt + dx^2 \tag{3.2}$$

for which now

$$x > vt\,, \tag{3.3}$$

$$r = x - vt = x' \tag{3.4}$$

this new variable defines, in the original Alcubierre metric, the proper spatial coordinate

$$dx = dx' + v\,dt$$

of the spaceship frame from which are observed the events in order to ensure a control communication.

Adopting the new coordinate

$$dx' = dx - v\,dt \tag{3.5}$$

and setting

$$S(r,t) = 1 - f(r,t)\,, \tag{3.6}$$

we may keep the metric (3.2) under the same form

$$(ds^2)_{\rm HS} = -\left[1 - v^2 S(r,t)^2\right] dt^2 - 2vS(r,t)\,dx'dt + dx^2. \tag{3.7}$$

We will refer to it as the *Hiscock metric* after William A. Hiscock [11]. It can be written as

$$(ds^2)_{\rm HS} = (g_{44})_{\rm HS}\,dt^2 + 2\,(g_{41})_{\rm HS}\,dx'dt + dx'^2 \tag{3.8}$$

with the covariant components of the fundamental tensor

$$\left.\begin{array}{l} (g_{44})_{\rm HS} = -\left(1 - v^2 S^2\right) \\ (g_{41})_{\rm HS} = (g_{14})_{\rm HS} = -\,vS \\ (g_{11})_{\rm HS} = (g_{22})_{\rm HS} = (g_{33})_{\rm HS} = 1 \end{array}\right\}. \tag{3.9}$$

The spaceship frame metric (3.7) is also expressed by

$$(ds^2)_{\text{HS}} = -\,H(r)\left(\frac{dt - vS}{H(r)}\,dx'\right)^2 + \frac{dx'^2}{H(r)}\,, \tag{3.10}$$

where

$$(g_{44})_{\text{HS}} = -\,H(r)\,,$$

we then introduce a new time coordinate

$$dt' = \frac{vS}{H(r)}\,dx', \tag{3.11}$$

which is manifestly the spaceship's proper time since $H(r)=1$ (thus $f=1$) as $r=0$.

At the same time, the coordinates are not asymptotically normalized. Indeed, for large r distant from the spaceship, $H(r)$ approaches $1-v^2$ rather than 1. One may solve the problem by defining yet one more set of coordinates

$$T' = \sqrt{1-v^2}\;t'\,, \qquad X = x'\sqrt{1-v^2}\;. \tag{3.12}$$

By examining the form of the metric (3.10), the coordinate system seems to be valid only for $r>0$, i.e. if $v<1$ as per (3.3).

However, when $v>1$ (superluminal velocity), there exists a coordinate singularity, that is, an event horizon at the location r_0 for the metric (3.10), such that

$$H(r_0) = 0$$

or

$$f(r_0) = 1 - \frac{1}{v}\,. \tag{3.13}$$

This horizon first appears for the occupants of the spaceship, who are unable to "see" beyond the distortion, and therefore cannot communicate with the outer universe.

§3.2. Reducing the energy

Based on the works produced by W. Hiscock, F. Loup, D. Waite and also E. Halerewicz et al. [12, 13], it has been proposed a particular metric which allows for the use of the warped region in order to "causally connect" the inside of the spaceship and the outside of the singular bubble region.

This generalized Hiscock metric (3.7) can also dramatically lower the negative energy requirements.

§3.2.1. The ESAA metric

By lowering the energy requirement, the proposed model intends to show that the Warp Drive metric is much more realistic than that originally shown by Pfenning and Ford.

We refer to this new space-time metric as *Ex Somnium Ad Astra* (ESAA), which literally translates as *From a Dream to the Stars* (Simon Jenks).

We are going to introduce the change $\rho = r_s$ of the variables. Independently of this change, the ESAA metric differs from (3.7) by the fundamental tensor whose covariant components are

$$\left.\begin{aligned} (g_{44})_{\text{ESAA}} &= -\left[\,N^2(\rho) - v_s\, S(\rho)^2\right] \\ (g_{41})_{\text{ESAA}} &= (g_{14})_{\text{ESAA}} = -\,v_s\, S(\rho) \\ (g_{11})_{\text{ESAA}} &= (g_{22})_{\text{ESAA}} = (g_{33})_{\text{ESAA}} = 1 \end{aligned}\right\}, \tag{3.14}$$

thus from these we readily note that the "time lapse" function is no longer equal to 1.

In cylindrical coordinates (following x), the ESAA metric is

$$(ds^2)_{\text{ESAA}} = -\left[N(\rho) - v_s(r)\, S(\rho)\right]^2 dt^2 - \\ - 2v_s\, S(\rho) + dx'^2 + dr^2 + r^2 d\phi^2. \tag{3.15}$$

Let us set

$$r = \rho \sin\theta\,, \qquad x' = r\cos\theta\,,$$

it is then easy to see that (3.15) becomes

$$(ds^2)_{\text{ESAA}} = \left[N^2(t,\rho) - v_s(t)\, S^2(\rho)\right] dt^2 + 2v_s(t)\, S(\rho)\, \cos\theta\, dt\, dr - \\ - 2v_s(t)\, S(\rho)\, \rho \sin\theta\, d\theta\, dt + dr^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta\, d\phi^2. \tag{3.16}$$

§3.2.2 Required energy

The energy density of the spaceship frame is given by

$$(T^{44})_{\text{ESAA}} = -\,\frac{v_s}{32\,\pi} \left(\frac{dS}{d\rho}\right)^2 \frac{(\sin\theta)^2}{N^4(\rho,t)}\,. \tag{3.17}$$

Clearly, an arbitrarily large N reduces the (negative) energy density requirement of the spaceship frame.

In our given coordinate system, the volume element is given by

$$dV = \rho^2 \sin\theta\, d\rho\, d\theta\, d\phi\,,$$

r_s	f	S	N
0	1	0	1.023
20	0.997	0.0023	3.428
50	0.5	0.5	2×10^{75}
100	4.5×10^{-5}	0.9999	1.0950

Table 1: Numerical estimates for the lapse function.

thus reinstating Newton's constant $\mathfrak{G}$ and c, the total energy required to sustain the distortion is finally given here by

$$E = -\int_0^\infty \left[\frac{v_s c^4}{12\mathfrak{G}} \left(\frac{dS}{d\rho}\right)^2 \frac{1}{N^4} \right] \rho^2 d\rho\,. \tag{3.18}$$

Another modification of the Alcubierre geometry has been suggested by Van den Broeck [14], in order to reduce the amount of needed negative energy.

The Van den Broeck metric is

$$ds^2 = -\,dt^2 + B^2(r_s)\big[dx - v_s(t)\,f(r_s)\,dt\big]^2 + dy^2 + dz^2,$$

where $B(r_s)$ is a twice differentiable polynomial such that its numerical value is $-1 < B(r_s) \leqslant 1+\alpha$ for $\Re' \leqslant r_s \leqslant \Re' + \Delta'$, and $B(r_s) = 1$ for $\Re' + \Delta' \leqslant r_s$ (here $\Re'$ is the radius of an internal "blown pocket" within the Alcubierre region with thickness Δ).

This modification keeps the surface area of the bubble itself microscopically small, while at the same time expanding the spatial volume inside the region caused by the factor α.

One can show that the energy density given by the tensor T_{44} is much lower than the one calculated by Alcubierre.

As an example, reinstating again the factor $c^2/\mathfrak{G}$, to get the kilogram units, for a bubble of $\Re = 100\,\text{m}$, the standard Alcubierre value for the total negative energy would be $E \approx -\,6.2\times10^{62}\, v_s$ Kg, which is theoretically enormous, but with the Van den Broeck solution ($v_s \approx 1$), this energy is reduced to 4.9×10^{30} Kg, that is a few solar masses: this shows that reasonable energy levels can be reached by investigating new models.

It is however difficult to establish energy level comparisons. This is because each model is characterized by different and newly introduced parameters.

In the case of the ESAA metric, we can, as an indication, compute some values for the functions f and S with the resulting lapse function

N, setting the initial values for the bump parameter as $\sigma = 0.1$ and bubble radius $\Re = 50$ m.

We first notice that in the warped regions ($r_s = \Re$), the lapse function N takes on very large values, which appears as a severe drawback, but interestingly, for $r_s > \Re$, the ESAA model yields back a lapse function $N \to 1$, which is in full accordance with the free fall condition (1.3).

§3.3. Causally connected spaceship

§3.3.1. The spaceship frame of reference

As we defined the Pfenning piecewise function (2.7) corresponding to the Alcubierre "top hat" function, we may establish the similar type of function with the time lapse N inserted

$$f(r_s)_{\text{p.c.}} = \begin{cases} 1 \ \text{ for } \ r_s < \Re - \frac{\Delta}{2}\,, \\ 1 - \left(\frac{1}{N}\right) r_s - \Re \ \text{ for } \ \Re - \frac{\Delta}{2} < r_s < \Re + \frac{\Delta}{2}\,, \\ 0 \ \text{ for } \ r_s > \Re + \frac{\Delta}{2}\,. \end{cases}$$

The "free fall" condition demands

$$N(r_s) = \begin{cases} 1 \ \text{ for } \ r_s < \Re - \frac{\Delta}{2}\,, \\ 1 \ \text{ for } \ r_s > \Re + \frac{\Delta}{2}\,. \end{cases}$$

The spaceship frame Hiscock-ESAA horizon is thus defined as

$$H(r_s) = \begin{cases} 1 \ \text{ for } \ r_s < \Re - \frac{\Delta}{2}\,, \quad H(r_s) > 0\,, \\ N^2 - \left(\frac{v_s}{N}\right)^2 \ \text{ for } \ r_s = \Re - \frac{\Delta}{2}\,, \quad H(r_s) > 0\,, \\ N^2 \ \text{ for } \ r_s = \Re\,, \quad H(r_s) > 0\,, \\ N^2 - \left(\frac{v_s}{N}\right)^2 \ \text{ for } \ r_s = \Re + \frac{\Delta}{2}\,, \quad H(r_s) > 0\,, \end{cases}$$

where we emphasize that N does not depend on the speed v_s.

Three cases are to be considered:

Subliminal velocities: For large values of N, the spaceship will always be connected to the domain from $r_s = 0$ (center of the spaceship) to the exterior part of the bubble $r_s = \Re + \frac{\Delta}{2}$, and since $H(r_s) > 0$, there is no horizon;

Luminal velocity: For the same domain, $H(r_s) = 0$, since $N = 1$ and $S(r) = 1$, a horizon will appear in front of the spaceship, which becomes causally disconnected from the part beyond the bubble.

Provided that N *is not a function of the speed and has been engineered at subluminal speeds*, it is always connected to the spaceship and the warped region $\int_{\Re-\Delta/2}^{\Re+\Delta/2}$ can be "controlled" by the "astronauts";

Superluminal velocities: The same argument applies here.

§3.3.2. A remote frame of reference

With the function $N(t, r_s)$, the Alcubierre metric is written

$$(d\tau^2)_{\text{ALN}} = (ds^2)_{\text{ALN}} = -N^2 dt^2 - \big[dx - v_s f(r_s, t)\, dt\big]^2$$

or

$$\begin{aligned}(ds^2)_{\text{ALN}} &= -\big(N^2 - v_s f^2(r_s, t)\big)\, dt^2 - 2v_s f(r_s, t)\, dt\, dx + dx^2 = \\ &= -M(r_s)\, dt^2 - 2v_s f(r_s)\, dx\, dt + dx^2,\end{aligned} \tag{3.19}$$

where $M(r_s) = N^2 - v_s^2 f(r_s)^2$.

We will refer to (3.19) as the *ESAA-Alcubierre metric*, as observed from a remote frame of reference.

The remote metric of Hiscock, analogous to (3.10), is thus given by

$$(ds^2)_{\text{ALN}} = -M(r_s)\, dt'^2 + \frac{N^2}{M(r_s)}\, dx^2,$$

lending

$$dt'^2 = -dt^2 - \frac{2v_s f(r_s)\, dx\, dt}{M(r_s)} + \frac{N^2 - M(r_s)}{M(r_s)^2}\, dx^2. \tag{3.20}$$

If $v_s < 1$ (subliminal), $M(r_s) > 0$ then the domain is causally connected to the spaceship's remote frame.

If $v_s = 1$ (luminal), $M(r_s) = 0$, a horizon appears for the remote frame.

If $v_s > 1$ (superluminal), $M(r_s) < 0$, a horizon appears somewhere between $\Re - \frac{\Delta}{2}$ and $r_s < \Re - \frac{\Delta}{2}$.

Using the continuous "top hat" function in (3.20) for the warped region of Pfenning $\big[\Re - \frac{\Delta}{2},\, \Re + \frac{\Delta}{2}\big]$, one obtains

$$M(r_s) = N^2 - h$$

with

$$h = \sqrt{1 - \big[v_s^2\, (r_s - \Re)^2\,\big] N^2(t, r_s)}\ .$$

Given that $N^2 \gg v_s f^2(r_s)$, then $M(r_s) > 0$ and the warped region will be always connected to the remote frame.

In other words, for large N, a signal can be sent by the spaceship to $r_s = \Re + \frac{\Delta}{2}$, and a signal sent by a remote observer can reach $r_s = \Re - \frac{\Delta}{2}$.

Therefore the region between

$$\Re - \frac{\Delta}{2} \leqslant r_s \leqslant \Re + \frac{\Delta}{2}$$

is observed from both frames, and may allow us to engineer the spaceship (speed control). Reverting now to the Alcubierre function

$$f(r_s, t) = \frac{\tanh\big[\sigma(r_s + \Re)\big] - \tanh\big[\sigma(r_s - \Re)\big]}{2\tanh(\sigma\Re)}\,,$$

we know that it is 1 in the spaceship and 0 far from it. There exists an open interval where $f(r_s, t)$ starts to decrease from 1 to 0, precisely where the negative energy is located.

In order to maintain the "free fall" condition (1.3), N should reduce to 1 in the spaceship and far from it outside the singular region.

In order to fulfill this condition, we suggest here the following form for N which differs from the formula (33) of [13]

$$N = \exp\Big(\tanh\big[\sigma(r_s - \Re)\big]^2\Big)\,. \tag{3.21}$$

This has the advantage of taking higher "peak" value near the spaceship where the excessive proper time $N d\tau$ is thus rapidly shortened as $r_s \to \Re$.

Chapter 4. The EGR-Like Picture

§4.1. A particular extended Lie derivative

Instead of considering the Alcubierre function f associated with a local Riemannian structure emerging from a background Euclidean space-time, we choose here to express f in the EGR-like formulation.

Unlike the classical theory, this singular region will now be distinguished from a non-flat background space-time i.e. a "weak" Riemannian background manifold, which is physically more appropriate.

Our aim is to find an additional energy decrease with a way to possibly avoid violating the weak energy condition.

We begin by defining an extended Lie derivative of g_{ab} that leads to a new extrinsic curvature.

Let us consider the infinitesimal coordinates shift

$$x'^a = x^a + N^a, \tag{4.1}$$

the relevant metric variation is classically given by

$$\delta g_{ab} = -\, g_{ac}\,\frac{\partial N^c}{\partial x^b} - g_{cb}\,\frac{\partial N^c}{\partial x_a} - \frac{\partial g_{ab}}{\partial x^c}\,N^c. \tag{4.2}$$

Furthermore it can be shown that [15]

$$\delta g_{ab} = (N_{a;b} + N_{b;a}) = \underset{N}{\mathrm{L}}\, g_{ab}\,. \tag{4.3}$$

When $\underset{N}{\mathrm{L}}\, g_{ab} = 0$, we have the Killing equations which preserve the metric (a condition referred to as infinitesimal isometry) under (4.1).

In the EGR theory, the metric undergoes an additional variation ζ upon (4.1) due to the covariant derivative of the metric, and we expect to find for the Killing equations the following expression

$$\underset{N}{\mathrm{L}}\, g_{ab} = \zeta\, g_{ab}\,. \tag{4.4}$$

We need now to define the explicit form of the infinitesimal variation ζ. To this effect we will first consider a vector l with components A^i such that

$$l^2 = g_{ik}\,A^i A^k$$

upon (4.1) this vector is varied by

$$l'^2 = (1+\zeta)\,l^2,$$

i.e.

$$dl^2 = \zeta\, l^2.$$

Obviously we have

$$dl^2 = (\mathrm{D}_c\, g_{ik})\,A^i A^k\,dx^c,$$

where, as stipulated in the EGR theory,

$$\mathrm{D}_c\, g_{ik} = \frac{1}{3}\left(J_k\, g_{ci} + J_i\, g_{ck} - J_c\, g_{ik}\right),$$

thus

$$dl^2 = l^2\, g^{ik}\,(\mathrm{D}_c\, g_{ik})\,dx^c$$

and so

$$\zeta = g^{ik}\,(\mathrm{D}_c\, g_{ik})\,dx^c$$

setting

$$g^{ik}\,(\mathrm{D}_c\, g_{ik}) = B_c$$

we write

$$\zeta\, g_{ab} = g_{ab}\,B_c\,dx^c.$$

Within a sufficient approximation, we may set

$$dx^c = N^c,$$

hence we define the "extended Lie derivative" of g_{ab} as

$$\underset{N'}{\mathrm{L}}\, g_{ab} \equiv \underset{N}{\mathrm{L}}\, g_{ab}\, B_c N^c, \tag{4.5}$$

where N' is the rescaled shift vector.

At this stage, we want to stress that the assumed extension is here always considered in a Riemannian scheme.

The definition (4.5) formally holds for a Lie derivative of g_{ab}, provided the last term is "likened" to a Riemannian correction.

Indeed a "non-Riemannian" Lie derivative (i.e. defined in the framework of the EGR theory) is not applicable, due to the algebraic nature of this operation.

The EGR theory however provides a justification as to the origin of the extra term in (4.5).

§4.2. Extended extrinsic curvature and associated energy density

We are now able to define the "extended" extrinsic curvature as

$$K'_{\alpha\beta} = (2N')^{-1} \left(\nabla_\alpha N'_\beta + \nabla_\beta N'_\alpha + \frac{\partial g_{\alpha\beta}}{\partial t} \right). \tag{4.6}$$

Accordingly, we still consider the classical field equations as inferred from the Hilbert-Einstein action

$$S = \int R \sqrt{-g}\; d^4x \,.$$

By doing so, we set forth a close one-to-one correspondence between the EGR scalar curvature $\mathsf{R} = R - \frac{1}{3}\left(\nabla_e J^e + \frac{1}{2} J^2\right)$ and the modified Riemannian scalar curvature R depicted in Riemannian geometry.

In this perspective, the equation (2.19) becomes here

$$C' = \frac{1}{6\pi} \left({}^{(3)}R' - K'_{\alpha\beta} K'^{\alpha\beta} + K'^2 \right). \tag{4.7}$$

Now we are going to generalize the Alcubierre metric by following the same pattern which has led to (2.20).

However, based on the extended formulation, we now choose

$$N'^1 = -\, v_s\, f(r_s)\,, \tag{4.8}$$

$$N'^2 = N^2, \qquad N'^3 = N^3 \tag{4.9}$$

and

$${}^{(3)}R' = {}^{(3)}R\,.$$

An immediate and important consequence appears when one observes the form of the expression

$$(C')_{\mathrm{AL}} = \frac{1}{16\pi}\left[{}^{(3)}R - K_{\alpha\beta}(N'^1, N)\, K^{\alpha\beta}(N'^1, N) + K^2(N'^1, N)\right]. \tag{4.10}$$

In contrast to the classical Alcubierre scheme, the non-vanishing initial Riemannian scalar curvature of the hypersurfaces may have now a significant impact on the negative energy density reduction.

In addition, the term K^2, which should not cancel off here, contributes even further to lowering this energy.

Discussion and Concluding Remarks

First observation: The expansion of the volume element $\theta = -K^\alpha_\alpha$ is attached to the bubble which it generates and is thus a local property;

Second observation: The free fall condition (1.3) requires obviously a flat space (flat Universe), instead of a Riemannian one.

However, in the EGR context, the non-vanishing scalar curvature ${}^{(3)}R$ may be also regarded here as sufficiently "local" with respect to the (quasi) Euclidean space as a whole, wherein the Eulerian observers are situated.

Indeed, if the three-volume of each hypersurface $t = const$ is extremalized, the condition $K = const$ results (see André Lichnérowicz [16] and also subsequently maximum slicing conditions by Yvonne Choquet Bruhat [17]).

It is then possible to impose this condition, with respect to using equation (1.16), to eliminate ${}^{(3)}R$ from the trace of equation (1.15): in this case it is shown that the lapse function can be taken to be $N \to 1$ as an asymptotic boundary condition, which leads to an asymptotically flat space-time.

This condition is physically satisfied when one considers the scale of distances in our observable Universe as compared to the bubble warping dimensions, so that (4.10) holds with an asymptotically flat universe wherefrom the distant observers are located.

Hence, we can always imagine a situation where stellar massive objects arranged in such a required configuration are coming into play, and where the influence of their curvature given by ${}^{(3)}R$ may then be used to balance the negative energy, which renders the Warp Drive compatible with the weak energy condition.

With these two observations our theory tends to run counter to the zero expansion Warp Drive suggested by José Natario [18].

Needless to say, all arguments regarding the piecewise function causality constraints detailed above are equally valid in our extended formulation.

Within the standard Alcubierre metric, it is however possible to avoid the problem of causally disconnecting the spaceship from the outer edge of the bubble.

A somewhat recent two dimensional metric concept has been proposed by Serge V. Krasnikov [19] in which the time for a round trip to a distant planet as measured by clocks located on the Earth can be made arbitrarily short.

To connect the Earth to the planet, a space-time extension of this metric leads to the creation of a "tube" wherein the space-time is flat, but the light cones are opened out so as to allow superluminal travel [20].

In some cases, these metrics are shown not to lead to the fatally closed time-like curves.

Appendix. Detailing a stellar round trip example according to Alcubierre

A1. Stellar journey

Consider two quasi-static planets A and B, which are apart from each other at a distance D in the Euclidean space-time.

A spaceship starts off on its own (self-propulsion) from A at an initial moment of time $t = t_A$, with a subluminal velocity $v < c$.

At a distance d away from A, $d \ll D$, the spaceship stops at a point where the bubble is being created, which then drags the spaceship towards the planet B, thus inducing a coordinate three-acceleration $\mathbf{a}$ that varies rapidly from $\mathbf{a} = 0$ to $\mathbf{a} = const \neq 0$.

Halfway, between A and B, the bubble is controlled so as to invert this acceleration from $\mathbf{a}$ to $-\mathbf{a}$.

As the absolute values of acceleration and deceleration are assumed equal, the spaceship will eventually be at rest at a distance d away from the planet B at the time the disturbance will disappear ($v_s = 0$) and the journey is further completed at a "physical" speed $v < c$.

The total coordinate time elapsed in the one-way trip from the planet A to the planet B is: $T = t_{\text{self-propulsion}} + t_{\text{bubble}}$. Had the acceleration been constant along the distance $(D-2d)$, we would have

$$(D-2d)=\frac{\mathbf{a}\,t^2_{\text{bubble}}}{2}\,,$$

where

$$t^2_{\text{bubble}}=\frac{2\,(D-2d)}{\mathbf{a}}\,.$$

In fact, during the accelerating stage of the bubble, we will have

$$^{(+)}t^2_{\text{bubble}}=\frac{(D-2d)}{\mathbf{a}}$$

and during the decelerating stage

$$^{(-)}t^2_{\text{bubble}}=\frac{(D-2d)}{\mathbf{a}}\,,$$

which in total yields

$$T=t_{\text{self-propulsion}}+\sqrt{\frac{2\,(D-2d)}{\mathbf{a}}}$$

that is

$$T=2\left(\frac{d}{v}+\sqrt{\frac{(D-2d)}{2\mathbf{a}}}\right).$$

A2. Deceleration stage

Remember that we considered planets A and B as static in a quasi-flat space. In this case $dx=dy=dz=0$. This means that their proper time is equal to their coordinate time (reinstating c): $t=\tau=\frac{x^4}{c}$.

The proper time τ measured in the spaceship, on the other hand, must take into account the Lorentz transformations

$$\tau_{\text{ship}}=2\left(\frac{d}{\gamma v}+\sqrt{\frac{(D-2d)}{2\mathbf{a}}}\right),$$

where

$$\gamma=\frac{1}{\sqrt{1-v^2/c^2}}\,.$$

If the radius $\Re$ of the bubble satisfies, as it should, $\Re\ll d\ll D$, one may admit the approximation

$$\tau\approx T\approx\sqrt{\frac{2D}{\mathbf{a}}}\,.$$

This clearly shows that T can be chosen as small as we like, by increasing the value of **a**.

As outlined by Alcubierre, since a round trip will only take twice as long, we can be back on the planet A after an arbitrarily short proper time, both from the point of view of an observer on board of the spaceship and from the point of view of an observer located on the planet.

The spaceship will then be able to travel much faster than the speed of light while remaining on a time-like trajectory (which is inside its local light-cone).

Submitted on December 10, 2009

1. Alcubierre M. The Warp Drive: hyper-fast travel within general relativity. *Classical and Quantum Gravity*, 1994, vol. 11, L73–L77.
2. Hawking S. W. The Large Scale Structure of Space-Time. Cambridge University Press, Cambridge, 1973, p. 88–92.
3. Marquet P. The EGR theory: an extended formulation of general relativity. *The Abraham Zelmanov Journal*, 2009, vol. 2, 148–170.
4. Kramer D., Stephani H., and Hertl E. Exact Solutions of Einstein's Field Equations. Cambridge University Press, Cambridge, 1980.
5. Arnowitt R. and Deser S. Quantum theory of gravitation: general formulation and linearized theory. *Physical Review*, 1959, vol. 113, no. 2, 745–750; Arnowitt R., Deser S., and Misner C. Dynamical structure and definition of energy in general relativity. *Physical Review*, 1959, vol. 116, no. 5, 1322–1330; Arnowitt R., Deser S., and Misner C. Canonical variables for general relativity. *Physical Review*, 1960, vol. 117, no. 6, 1595–1602.
6. Kuchař K. Canonical methods of quantization. *Quantum Gravity 2. A Second Oxford Symposium*, Clarendon Press, Oxford, 1981, 329–374.
7. MacCallum M. A. H. Quantum cosmological models. *Quantum Gravity. Proceedings of the Oxford Symposium.* Clarendon Press, Oxford, 1975, 174–218.
8. Stewart J. M. Numerical relativity. In: *Classical General Relativity*, Cambridge University Press, Cambridge, 1982, 231–262.
9. Ford L. H. and Pfenning M. J. The unphysical nature of "warp drive". *Classical and Quantum Gravity*, 1997, vol. 14, no. 7, 1743–1751; Cornell University arXiv: gr-qc/9702026.
10. Wald R. General Relativity. University of Chicago Press, Chicago, 1984.
11. Hiscock W. A. Quantum effects in the Alcubierre warp-drive spacetime. *Clasical and Quantum Gravity*, 1997, vol. 14, no. 11, L183–L188; Cornell University arXiv: gr-qc/9707024.
12. Loup F., Waite D., and Halerewicz E. Reduced total energy requirement for a modified Alcubierre warp drive space-time. Cornell University arXiv: gr-qc/0107097.
13. Loup F., Held R., Waite D., Halerewicz E. Jr., Stabno M., Kuntzman M., Sims R. A causally connected superluminal Warp Drive spacetime. Cornell University arXiv: gr-qc/0202021.

14. Van den Broeck C. A "warp drive" with more reasonable total energy requirements. *Classical and Quantum Gravity*, 1999, vol. 16, no. 12, 3973–3979; Cornell University arXiv: gr-qc/9905084.

15. Landau L. D. et Lifshitz E. M. Théorie des Champs. Traduit du Russe par E. Gloukhian, Éditions de la Paix, Moscou, 1964.

16. Lichnérowicz A. L'integration des equations de la gravitation relativiste et le probleme des n corps. *Journal de Mathématiques Pures et Appliquées*, 1944, tome 23, 37–63.

17. Choquet-Bruhat Y. Maximal submanifolds and manifolds with constant mean extrinsic curvature of a Lorentzian manifold. *Annali della Scuola Normale Superiore di Pisa, Classe di Science*, 1976, tome 3, no. 3, 373–375.

18. Natario J. Warp drive with zero expansion. *Classical and Quantum Gravity*, 2002, vol. 19, no. 6, 1157–1165.

19. Krasnikov S. V. Hyperfast travel in general relativity. *Physical Review D*, 1998, vol. 57, no. 8, 4760–4766; Hyper fast interstellar travel in gneral relativity. Cornell University arXiv: gr-qc/9511068.

20. Everett A. E. and Roman T. A. Superluminal subway: the Krasnikov tube. *Physical Review D*, 1997, vol. 56, no. 4, 2100–2108.

Vol. 2, 2009 **ISSN 1654-9163**

THE ABRAHAM ZELMANOV JOURNAL

The journal for General Relativity, gravitation and cosmology

TIDSKRIFTEN ABRAHAM ZELMANOV

Den tidskrift för allmänna relativitetsteorin, gravitation och kosmologi

Editor (redaktör): Dmitri Rabounski
Secretary (sekreterare): Indranu Suhendro

The Abraham Zelmanov Journal is a non-commercial, academic journal registered with the *Royal National Library of Sweden.* This journal was typeset using LaTeX typesetting system. Powered by Ubuntu Linux.

The Abraham Zelmanov Journal är en ickekommersiell, akademisk tidskrift registrerat hos *Kungliga biblioteket.* Denna tidskrift är typsatt med typsättningssystemet LaTeX. Utförd genom Ubuntu Linux.