Geodesics and Finslerian Equations in the EGR Theory

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Abstract: In the framework of the EGR theory, introduced recently by the author as a non-Riemannian extension of the General Theory of Relativity, the geodesic equations for a free neutral particle have the same form as those in Riemannian geometry except that they describe the particle's motion together with its own gravitational field, thus forming a global dynamical massive entity. In this paper, we show that in the case of a charged mass moving in an external electromagnetic field, the gravitational field of the global mass interacts with the electromagnetic potential through its current density. This interaction process must necessarily take a place in order for the global charge's lines of motion to satisfy a differential Finslerian system of equations whose form is similar to that of Riemannian geometry, as is the case for the neutral particle's geodesics. This result represents further evidence that the EGR model is an appropriate description of the mass and its subsequent gravitational field as a whole.

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Chapter 1. The Neutral Mass

§1.1. The EGR energy-momentum tensor. The EGR theory (Extended General Relativity) was introduced recently by the author [1] as a non-Riemannian extension of the General Theory of Relativity. Let us first recall the EGR source-free field equations

$$(G_{ab})_{\text{EGR}} = (R_{ab})_{\text{EGR}} - \frac{1}{2}\left[g_{ab}(R)_{\text{EGR}} - \frac{2}{3}\, J_{ab}\right] = \varkappa\,(T_{ab})_{\text{field}} \tag{1.1}$$

with

$$J_{ab} = \partial_a J_b - \partial_b J_a\,, \tag{1.2}$$

where $\varkappa$ is Einstein's constant, and $c=1$. Here $(T_{ab})_{\text{field}}$ is the energy-momentum tensor of the EGR persistent field related to the tensor density $\sqrt{-g}\,(T^{ab})_{\text{field}} = (\mathcal{T}^{ab})_{\text{field}}$, which is determined through the equations

$$(\mathcal{T}^a_b)_{\text{field}} = \frac{1}{2\varkappa}\left[\partial_b \Gamma^e_{df}\,\frac{\partial\mathcal{H}}{\partial(\partial_a\Gamma^e_{df})} - \delta^a_b\,\mathcal{H}\right]. \tag{1.3}$$

The invariant density $\mathcal{H}$ is given by $\mathcal{H} = \mathcal{R}^{ab}R_{ab}$ with the EGR Ricci tensor density $\mathcal{R}^{ab} = R^{ab}\sqrt{-g}$.

This background EGR field is assumed to be ever-present in vacuum. When matter is present, our previous studies [1–3] have led us to infer that the particle's (bare) mass density ρ is slightly modified, thus denoted hereinafter by ρ^*. The global quantity ρ^* is that part of the region surrounding the mass density, where Riemannian geometry increasingly dominates over the global one when asymptotically approaching the "bare" mass density ρ: it eventually becomes the single true geometry in the quasi "contact" situation. The reduction of the geometry in the immediate vicinity of the mass, can best be depicted by the transition of the surrounding EGR persistent field tensor density to the pseudo-tensor density ill-defined by Landau and Lifshitz, which conventionally describes the massive gravitational field

$$(\mathcal{T}^a_b)_{\text{LL}} = \frac{1}{2\varkappa}\left[\partial_b\,\mathcal{G}^{ec}\,\frac{\partial\mathcal{L}_{\text{E}}}{\partial(\partial_a\,\mathcal{G}^{ec})} - \delta^a_b\,\mathcal{L}_{\text{E}}\right], \quad \mathcal{G}^{ab} = \sqrt{-g}\,g^{ab}, \tag{1.4}$$

where $\mathcal{L}_{\text{E}} = \sqrt{-g}\,g^{ab}\left(\{^{\,e}_{ab}\}\{^{\,d}_{de}\} + \{^{\,d}_{ae}\}\{^{\,e}_{bd}\}\right)$ is the Einstein Lagrangian density.

Therefore, the EGR theory enables to regard the quantity ρ^* as a generalized mass density including its own gravitational field, thus forming a single dynamical entity. Naturally, the correction brought to

Riemannian geometry is assumed to be weak. Hence, we write down the global energy-momentum tensor as

$$(T^{ab})_{\rm EGR} = \rho^*(u^a u^b)_{\rm EGR} \tag{1.5}$$

or

$$(T^{ab})_{\rm EGR} = \rho\,(u^a u^b)_{\rm EGR} + (t^{ab})_{\rm grav}\,, \tag{1.6}$$

where $(t^{ab})_{\rm grav}$ is the tensor of the gravitational field associated with the mass density, which classically corresponds (in Riemannian geometry) to the Landau-Lifshitz pseudo-tensor density (1.4).

The tensor $(t^{ab})_{\rm grav}$ is antisymmetric in accordance with the form of the EGR Einstein tensor $(G^{ab})_{\rm EGR}$ (1.1), and so is implicitly the tensor $(T^{ab})_{\rm EGR}$ (1.6).

The EGR Einstein tensor $(G_{ab})_{\rm EGR}$ obeys the conservation law

$$\left\{(R^b_a)_{\rm EGR} - \frac{1}{2}\left[\delta^b_a (R)_{\rm EGR} - \frac{2}{3}\, J^b_a\right]\right\}_{\prime,b} = 0\,. \tag{1.7}$$

Unlike in Riemannian geometry wherein covariant derivatives are constructed with the Christoffel symbols, the condition (1.7) utilizes the generally covariant derivatives $\prime,$ (also denoted here by the symbol D) built from the global connection [1]

$$\Gamma^d_{ab} = \left\{{d \atop ab}\right\} + (\Gamma^d_{ab})_J = \left\{{d \atop ab}\right\} + \frac{1}{6}\left(\delta^d_a J_b + \delta^d_b J_a - 3\,g_{ab}\,J^d\right). \tag{1.8}$$

Therefore, in the absence of matter, the persistent field tensor we denote as $(T_{ab})_{\rm field}$ should be conserved according to (1.7)

$$\left[(T^b_a)_{\rm field}\right]_{\prime,b} = 0\,. \tag{1.9}$$

For the “ massive” case, we have

$$\left[(T^b_a)_{\rm EGR}\right]_{\prime,b} = \left[\,\rho\,(u^b u_a)_{\rm EGR} + (t^b_a)_{\rm grav}\right]_{\prime,b} = 0 \tag{1.10}$$

or, written equivalently,

$$\left[(T^b_a)_{\rm EGR}\right]_{\prime,b} = \left[\,\rho^*(u^b u_a)_{\rm EGR}\right]_{\prime,b} = 0\,. \tag{1.11}$$

§1.2. The EGR geodesics. A free neutral particle with mass m_0 classically follows a time-like geodesic according to the equation

$$\frac{d^2x^b}{ds^2} + \left\{{b \atop cd}\right\}\frac{dx^c}{ds}\frac{dx^d}{ds} = 0 \tag{1.12}$$

defined in a 4-Riemannian manifold equipped with a metric satisfying

$$g_{ab}\, u^a u^b = g^{ab}\, u_a u_b = 1\,, \tag{1.13}$$

where $u^a = \frac{dx^a}{ds}$ is the corresponding unit 4-vector (the world-velocity of the particle).

Following the EGR theory, inspection shows that the time-like geodesic equations shall have the same form

$$\left(\frac{d^2x^b}{ds^2}\right)_{\text{EGR}} + \Gamma^b_{cd}\left(\frac{dx^c}{ds}\frac{dx^d}{ds}\right)_{\text{EGR}} = 0\,. \tag{1.14}$$

Besides, the EGR world-velocity is slightly modified by the presence of the linear (non-square) form

$$dJ = f\,(J_b)\, dx^b, \tag{1.15}$$

so that the 4-velocity u^a becomes

$$(u^a)_{\text{EGR}} = \frac{dx^a}{\sqrt{ds^2 + dJ}}\,. \tag{1.16}$$

We also assume here that

$$g_{ab}\,(u^a u^b)_{\text{EGR}} = g^{ab}\,(u_a u_b)_{\text{EGR}} = 1\,. \tag{1.17}$$

Chapter 2. The Charged Mass

§2.1. Charged density in an electromagnetic field. With further contribution due to an external electromagnetic field, namely the Maxwell tensor F_{ab}, the geodesics of a particle with mass m_0 and charge e, are generated by the Finslerian curves which are known to be solutions of the Riemannian differential system

$$u^a\,\nabla_a u_b = \frac{e}{m_0}\,F_{ba}\,u^a = \frac{\mu}{\rho}\,F_{ba}\,u^a, \tag{2.1}$$

where ρ and μ are, respectively, the mass density and the charge density of the particle. An alternative form of (2.1) is given by the well-known formula

$$\frac{d^2x^b}{ds^2} + \left\{{}^{\,b}_{cd}\right\}\frac{dx^c}{ds}\frac{dx^d}{ds} = \frac{\mu}{\rho}\,F^b_a\,\frac{dx^a}{ds}\,, \tag{2.2}$$

where the current vector is given by

$$j^a = \mu u^a. \tag{2.3}$$

The charged particle is said to satisfy a *Finslerian flow line*.

Classically, the general electromagnetic field energy-momentum tensor $(T_{ab})_{\text{elec}}$ is inferred from the Lagrangian

$$L = -\frac{1}{16\pi}\, F^{ab} F_{ab} - j_a A^a. \tag{2.4}$$

Henceforth, we use the Heaviside system of units where $\frac{1}{4}$ is substituted to the Gauss system $\frac{1}{16\pi}$.

As is well-known, the potential $A^a(x^a)$ is not a directly observable quantity, but is determined within a gradient

$$A'^a = A^a + \partial_a \psi\,. \tag{2.5}$$

Therefore it is customary to adopt a special gauge, which may be the *Lorentz gauge*. For the consistency of the theory, we keep this type of gauge throughout the text.

The tensor $(T_{ab})_{\text{elec}}$ is symmetrized, so as to yield

$$(T^{ab})_{\text{elec}} = \frac{1}{4}\, g^{ab} F_{cd} F^{cd} + F^{am} F_{m\cdot}^{\cdot\, b} + g^{ab} j_m A^m - j^a A^b. \tag{2.6}$$

However, the presence of sources violates (in general) the gauge invariance and also prevents this tensor from obeying the conservation law. This is why, in order to fit in the (symmetric) Einstein equations, one adds the symmetrized tensor (2.6) without source on the right-hand side of the Einstein-Maxwell field equations as

$$G_{ab} = \varkappa \left[\, \rho\, u_a u_b + (T_{ab})_{\text{elec}} \right]. \tag{2.7}$$

This somewhat arbitrary "adjustment" is true evidence of the rather awkward electromagnetic contribution to the classical field equations. In this sense, Riemannian geometry appears to be unable to thoroughly describe electrodynamics in the standard general relativistic theory. The problem can be cured by using the non-Riemannian connection as applied in the EGR theory, where the Einstein tensor is no longer symmetric. This intrinsic property allows for a straightforward and natural use of the canonical energy-momentum tensor of the electromagnetic field in the EGR Einstein field equations. As will be shown in §2.2 this canonical tensor is readily derived from a generalized Lagrangian density obtained in an analogous way as that used to deduce the EGR field equations.

§2.2. The EGR electromagnetic current density. Introducing the 4-potential A_a, the Maxwell tensor is written as

$$F_{ab} = \mathrm{D}_a A_b - \mathrm{D}_b A_a\,. \tag{2.8}$$

We proceed in strict analogy to the EGR stationary principle and set a Lagrangian density defined from the tensor and vector densities

$$\mathcal{F}^{ab} = \frac{\partial \mathcal{L}}{\partial F_{ab}}\,, \qquad \mathcal{I}^{a} = \frac{\partial \mathcal{L}}{\partial A_{a}}\,, \tag{2.9}$$

$$\mathcal{F}^{ab} = \sqrt{-g}\,F^{ab}, \tag{2.10}$$

$$\mathcal{I}^{a} = \sqrt{-g}\,I^{a}. \tag{2.11}$$

The varied action is then given by

$$\delta\mathcal{S} = \delta \int \mathcal{L}\,d^4x = 0\,. \tag{2.12}$$

For a variation δA_a, we further obtain

$$\delta\mathcal{S} = \int \left(\frac{1}{2}\frac{\partial \mathcal{L}}{\partial F_{ab}}\,\delta F_{ab} + \frac{\partial \mathcal{L}}{\partial A_{a}}\,\delta A_a \right) d^4x = 0\,, \tag{2.13}$$

while the variation of $\mathcal{L}$ is expressed by

$$\int \left(\frac{1}{2}\,\mathcal{F}^{ab}\delta F_{ab} + \mathcal{I}^{a}\,\delta A_a \right) d^4x = 0\,. \tag{2.14}$$

We integrate (2.14) by parts

$$\int \left[\frac{1}{2}\,\mathcal{F}^{ab}\left(\partial_a \delta A_b - \partial_b\,\delta A_a\right) + \mathcal{I}^{a}\,\delta A_a \right] d^4x =$$

$$= -\int \partial_b \left(\mathcal{F}^{ab}\delta A_a\right) d^4x + \int \left(\partial_b\,\mathcal{F}^{ab} + \mathcal{I}^{a}\right) \delta A_a\, d^4x = 0\,. \tag{2.15}$$

If the variations of A_a are zero on the integration boundary, the first integral yields no contribution. Then $\delta\mathcal{S} = 0$ implies

$$\partial_b\,\mathcal{F}^{ab} = -\,\mathcal{I}^{a}. \tag{2.16}$$

We clearly see that (2.8) and (2.16) represent the second group of Maxwell's equations given by the current density

$$\mathcal{I}^{a} = \sqrt{-g}\,I^{a}. \tag{2.17}$$

With a dynamical mass bearing its own gravitational field and having charge density μ, the global electromagnetic current density is obviously given by

$$I^{a} = \mu\,(u^a)_{\text{EGR}}\,, \tag{2.18}$$

which is collinear with the unit vector $(u^a)_{\text{EGR}}$.

Since the EGR description includes the gravitational field of the charged mass, it is natural to assume that this field interacts with the external electromagnetic field through the coupling between the current I_a and the potential A_a. This can be achieved by taking into account the term represented by the tensor

$$(T_{ab})_{\text{int}} = A_a I_b \,, \tag{2.19}$$

which is equivalent to saying that the dynamical mass ρ^* is affected by the interaction as follows

$$\rho^* (u_a u_b)_{\text{EGR}} \to \rho^* (u_a u_b)_{\text{EGR}} + (T_{ab})_{\text{int}} = (\rho^*)_{\text{int}} (u_a u_b)_{\text{EGR}} \,. \tag{2.20}$$

This assumption will find its full justification in §3.2.

Chapter 3. The EGR Differential Equations

§3.1. The EGR energy-momentum tensor of an electromagnetic field. Classically, the Lagrangian density displaying the current-potential coupling, and the Lagrangian itself are written as

$$\mathcal{L} = \frac{1}{2}\,\mathcal{F}^{ab} F_{ab} - A_a \mathcal{I}^a, \qquad L = \frac{1}{2}\,F^{ab} F_{ab} - A_a I^a. \tag{3.1}$$

Because we use the Heaviside system of units (see the note below formula 2.4 in page 94), the Lagrangian has the form

$$L = -\frac{1}{4}\,F^{ab} F_{ab} - A_a I^a. \tag{3.2}$$

The canonical energy-momentum tensor density $(\mathcal{E}^{ab})_{\text{EGR}}$ of the electromagnetic field is inferred from the Lagrangian density $\mathcal{L}$ (3.1). This (antisymmetric) tensor density has the usual generic form

$$(\mathcal{E}^{ab})_{\text{EGR}} = \left[\frac{\partial \mathcal{L}}{\partial(\partial_a A_m)}\right] \partial^b A_m - g^{ab}\mathcal{L}, \tag{3.3}$$

which has also a tensor counterpart such that

$$\sqrt{-g}\;\Theta^{ab} = (\mathcal{E}^{ab})_{\text{EGR}} \,. \tag{3.4}$$

Since the EGR Einstein tensor $(G_{ab})_{\text{EGR}}$ is not symmetric, it is thus most natural to apply the canonical energy-momentum tensor Θ_{ab} right away on the right-hand side of the EGR field equations. Therefore, in the case of a massive charged matter, the EGR field equations can be written with the electromagnetic field tensor as

$$(G_{ab})_{\text{EGR}} = \varkappa \left[\rho^* (u_a u_b)_{\text{EGR}} + \Theta_{ab} \right]. \tag{3.5}$$

The charged mass density is now represented by the global tensor

$$\begin{aligned}(T_{ab})_{\text{EGR}} &= \rho \, (u_b u_a)_{\text{EGR}} + (t_{ab})_{\text{grav}} + \Theta_{ab} = \\ &= \rho^* (u_b u_a)_{\text{EGR}} + \Theta_{ab} \, .\end{aligned} \tag{3.6}$$

Obviously, the persistent field tensor $(T_{ab})_{\text{field}}$ does not appear explicitly on the right-hand side since we are here considering the "global massive" case. Also, the global mass ρ^* density is unaffected by the electromagnetic interaction (2.20) for the latter coupling is already included in the canonical tensor Θ_{ab}.

With the well-known classical identity

$$\frac{\partial \, (F^{kl} F_{kl})}{\partial (\partial_a A_m)} = 4 F^{am}, \tag{3.7}$$

we obtain the canonical tensor Θ_{ab}, which is given in the EGR formulation by the formula

$$\Theta^{ab} = \frac{1}{4} \, g^{ab} F_{kl} F^{kl} - F^{am} \, \mathrm{D}^b A_m + g^{ab} I_m A^m \tag{3.8}$$

expressed with the EGR current density $I^m = \mu \, (u^m)_{\text{EGR}}$ (2.18).

Using the tensor relations deduced from the equations of motion (2.16), and taking into account the antisymmetry of F_{ab}

$$\mathrm{D}_a F^{ba} = I^b, \tag{3.9}$$

we obtain a formula for the 4-divergence of Θ_{ab}. It is

$$\begin{aligned}\mathrm{D}_a \Theta^{ab} &= \frac{1}{2} \left(\mathrm{D}^b F_{kl} \right) F^{kl} - \left(\mathrm{D}_a F^{am} \right) \mathrm{D}^b A_m - F^{am} \mathrm{D}_b \mathrm{D}^b A_m + \\ &\quad + \left(\mathrm{D}^b I_m \right) A^m + I_m \mathrm{D}^b A^m = \\ &= -\frac{1}{2} \, \mathrm{D}^b \left(\mathrm{D}_k A_l + \mathrm{D}_l A_k \right) F^{kl} + \left(\mathrm{D}^b I_m \right) A^m,\end{aligned} \tag{3.10}$$

which, due to the antisymmetry of F_{kl}, obviously reduces to

$$\mathrm{D}_a \Theta^{ab} = (\mathrm{D}^b I_m) \, A^m. \tag{3.11}$$

We note in passing that the canonical tensor is conserved in the absence of electric current, which will be written as $(\Theta_{ab})_{\text{free}}$.

In the latter case, the gauge change

$$A'_a \to A_a - \partial_a \psi \tag{3.12}$$

finally yields

$$(\Theta'^{ab})_{\text{free}} = (\Theta^{ab})_{\text{free}} - \mathrm{D}_k \,(F^{ak}\, \partial^b \psi) \tag{3.13}$$

having $\mathrm{D}_k F^{ka} = I^a = 0$ taken into account.

Hence, $(\Theta^{ab})_{\text{free}}$ is not gauge invariant, but the second divergence term yields no contribution upon integration, and thus $(\Theta^{ab})_{\text{free}}$ is here a conserved quantity. Therefore, this (antisymmetric) canonical source-free tensor should be the appropriate candidate for the EGR field equations (3.5), provided we use the modified global mass density $(\rho^*)_{\text{int}}$.

In place of (3.6), we eventually write the equivalent formula

$$(T_{ab})_{\text{EGR}} = (\rho^*)_{\text{int}} (u_a u_b)_{\text{EGR}} + (\Theta_{ab})_{\text{free}}\,. \tag{3.14}$$

Unlike in Riemannian geometry, we clearly see that the EGR formulation allows us to include the electromagnetic source contribution represented by $(\rho^*)_{\text{int}}$, in the EGR Einstein-Maxwell equations.

In the absence of matter, the EGR energy-momentum tensor of the pure electromagnetic field is simply

$$(T_{ab})_{\text{EGR}} = (T_{ab})_{\text{field}} + (\Theta_{ab})_{\text{free}}\,. \tag{3.15}$$

§3.2. The EGR differential equations for the density flow lines of a charged mass. Our final aim is to find a differential system satisfied by the global charge, whose form is similar to the Riemannian system (2.2), as is the case for a neutral mass. To this effect, we first revert to the global energy-momentum as written in (3.6), for which the conservation law is given by

$$\left[\, \rho^* (u^b u_a)_{\text{EGR}} + \Theta^b_a \right]_{\prime ,\, b} = 0\,. \tag{3.16}$$

We introduce the vector K_b defined by

$$\rho^* K_b = \mathrm{D}_a \Theta^a_b = (\mathrm{D}_b I_m)\, A^m. \tag{3.17}$$

For that, we write the right-hand side as follows

$$\mathrm{D}_b (I_m A^m) - I_m \mathrm{D}_b A^m = (\mathrm{D}_b I_m)\, A^m, \tag{3.18}$$

and noting that

$$I_m \mathrm{D}_b A^m = \frac{1}{2}\, I_m (\mathrm{D}_b A^m - \mathrm{D}^m A_b) \tag{3.19}$$

the conservation conditions for the global tensor take the form

$$\mathrm{D}_a\left[\,\rho^*(u^a u_b)_{\mathrm{EGR}}\right] = -\,\rho^* K_b =$$
$$= \mathrm{D}_b\left(I^m A_m\right) - \frac{1}{2}\,I_m\left(\mathrm{D}_b A^m - \mathrm{D}^m A_b\right). \quad (3.20)$$

Taking into account the formula

$$(u^b)_{\mathrm{EGR}}\,\mathrm{D}_a (u_b)_{\mathrm{EGR}} = 0\,, \quad (3.21)$$

which follows from differentiating (1.17), we find, after multiplying through (3.20) by $(u^b)_{\mathrm{EGR}}$,

$$\mathrm{D}_a\left[\,\rho^*(u^a)_{\mathrm{EGR}}\right] = -\,\rho^* K_a (u^a)_{\mathrm{EGR}}\,. \quad (3.22)$$

The continuity equation is thus expressed by

$$\mathrm{D}_a\left[\,\rho^*(u^a)_{\mathrm{EGR}}\right] = -\,\mu\,(u^a)_{\mathrm{EGR}}\times$$
$$\times\left\{\mathrm{D}_a\left[(u_m)_{\mathrm{EGR}}A^m\right] - \frac{1}{2}\,(u_m)_{\mathrm{EGR}}\big(\mathrm{D}_a A^m - \mathrm{D}^m A_a\big)\right\}. \quad (3.23)$$

After some simplifications, we arrive at the differential system determining the flow lines of the charged particle

$$(u^a)_{\mathrm{EGR}}\,\mathrm{D}_a (u_b)_{\mathrm{EGR}} = \left[\,\delta^a_b - (u^a u_b)_{\mathrm{EGR}}\right]\times$$
$$\times\,\frac{\mu}{\rho^*}\left\{-\mathrm{D}_a\left[(u^m)_{\mathrm{EGR}}A_m\right] + \frac{1}{2}\,F_{am}(u^m)_{\mathrm{EGR}}\right\}. \quad (3.24)$$

Now, if we assume that the dynamical mass density ρ^* interacting with the potential A_m is modified so that

$$-\,K_b(\rho^*)_{\mathrm{int}} = \mathrm{D}_a\left[(\rho^*)_{\mathrm{int}}(u^a u_b)_{\mathrm{EGR}}\right] = -\mu\,\mathrm{D}_b\left[(u_m)_{\mathrm{EGR}}A^m\right], \quad (3.25)$$

we eventually obtain

$$(u^a)_{\mathrm{EGR}}\,\mathrm{D}_a (u_b)_{\mathrm{EGR}} = \left[\,\delta^a_b - (u^a u_b)_{\mathrm{EGR}}\right]\frac{1}{2}\,F_{am}(u^m)_{\mathrm{EGR}}\,. \quad (3.26)$$

These equations are to be compared to the Riemannian differential system

$$u^a\,\nabla_a u_b = \left(\delta^a_b - u^a u_b\right)\frac{\mu}{\rho}\,F_{am}u^m, \quad (3.27)$$

which reduces to the well-known classical equations $u^a\,\nabla_a u_b = \frac{\mu}{\rho}\,F_{ba}u^a$ (2.1) since F^{am} is antisymmetric.

In other words, imposing the above-mentioned conditions, the Finslerian trajectory of the global charged density will now satisfy the differential system

$$\left(\frac{d^2x^b}{ds^2}\right)_{\rm EGR} + \Gamma^b_{cd}\left(\frac{dx^c}{ds}\frac{dx^d}{ds}\right)_{\rm EGR} = \frac{1}{2}\,\frac{\mu}{(\rho^*)_{\rm int}}\,F^b_a\left(\frac{dx^a}{ds}\right)_{\rm EGR}. \qquad (3.28)$$

Within the numerical factor $\frac{1}{2}$, this EGR formulation is formally similar to the differential system of Riemannian geometry satisfied by the charged mass density trajectory according to the classical General Relativity.

Conclusion. Upon imposing the Lorentz gauge, we are able to generalize some basic principles of electrodynamics via the EGR theory. In the EGR formulation, three main results readily emerge:

1) Unlike in Riemannian geometry, the (antisymmetric) canonical electromagnetic energy-momentum tensor (3.8), as inferred from the Lagrangian (3.2), can be readily used in the EGR field equations, without post-symmetrization adjustment;
2) The dynamical global charged mass (current) interacting with the electromagnetic field implicitly appears in the EGR field equations. This result is impossible to express in Riemannian geometry (classical General Relativity), which stands so far as a profound loss of generality in the metric theory;
3) With the 2nd condition outlined above in this list, we are eventually able to infer the differential system (3.28) obeyed by the global charged mass, which is formally similar to the differential system (2.2) introduced according to Riemannian geometry, a similarity already existent between the Riemannian and EGR geodesics for the neutral mass, as given by (1.12) and (1.14). This last result gives us further evidence to substantiate the EGR model as representing the motion of a mass dynamically bearing its own gravitational field.

In conclusion, therefore, a last important point should be outlined here. Either the geodesic equations (1.14) for a neutral particle, or the Finslerian equations for a charged particle (3.28) (each system with its own corresponding gravitational field), does not distinguish antimatter from matter. The EGR model can, however, be adequately used to interpret the fermionic-antifermionic symmetry as postulated by Louis de Broglie, and generalized in [4].

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