Traversable Space-Time Wormholes Sustained by the Negative Energy Electromagnetic Field

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Abstract: We consider the Thorne-Morris static space-time wormhole, sustained by the so-called exotic matter which may produce a huge space-time distortion to achieve hyper-fast interstellar travel. Modifying this metric, we suggest such a particular type of matter by means of the negative electromagnetic energy density. This possibility relies on Maxwell's equations, which are applied to time-varying electromagnetic fields, and synchronously time-varying electromagnetic 4-current densities. By choosing the proper phase displacements, the time component of the electromagnetic stress-energy tensor displays a negative energy density in part produced by the interacting electromagnetic potential superimposed onto the current density. The positive energy part of this tensor does not make a contribution, since it is confined at the outer vicinity of the wormhole.

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Introduction. During the last decades, many published papers were devoted to space-time shortcuts among whom the *space-time wormhole model* introduced by Thorne and Morris [1] can be selected. This concept is very similar to the *Einstein-Rosen bridge* [2, p.198], but instead, the wormhole connects two distinct Universes, referred to as the *lower* and *upper* worlds. To be physically sustained, it is well-known that this model requires a negative energy density which implies the existence of *exotic matter* (as coined by Kip Thorne) and which classically violates all energy conditions [3]. Apart from the *averaged null energy condition* (ANEC), and the *averaged weak energy condition* (AWEC), we will be also interested in a global energy condition [4] which is referred to as the *volume integral quantifier* and which is linked to the Visser-Kar-Dadhich (VKD) wormholes [5]. In this approach, the total (exotic) energy is considered by performing a specific integration with respect to the matter proper volume element, and the amount of the (global) energy condition violations is measured when the integral becomes negative. This class of energy violations provides useful information and in particular, it determines the optimum choice of the *thickness* of the exotic matter layer threading the throat of the wormhole.

In Chapter 1, we first review the standard Lorentzian wormhole model, and we modify the metric in order to include a particular breakdown in the shape function. This breakdown accounts for two regions: the *inner* throat itself and an *outer* surrounding compact shell that is asymptotically fading away in order to merge with the quasi-Minkowskian Universe. Splitting the shape function into an *inner* region and an *outer* close shell does not affect the general wormhole physics. In Chapter 2, we investigate the possibility of using a variable electromagnetic field which interacts with a time-varying electric current, so that the resulting energy-momentum tensor splits up into a positive part and a negative part. This splitting is then required to correspond to the wormhole shape function breakdown, so that the positive electromagnetic free field contribution can be generated in the shell, i.e. *outside* the throat, while keeping the negative energy part *inside* to provide the necessary exotic matter, without violating the energy conditions.

Chapter 1. The Static Lorentzian Wormhole

§1.1. Definitions: the basic metric. In the signature +2, let us consider a generic static space-time

$$ds^2 = g_{ab}\,dx^2 dx^b = -e^{2\Phi(r)}dt^2 + g_{\mu\nu}\,dx^\mu dx^\nu,$$

where we set $G = c = 1$. Latin indices (a, b) run from 0 to 3; Greek indices $(\mu, \nu,)$ run from 1 to 3.

We then recall here the general static spherically symmetric *wormhole solution*

$$ds = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right), \qquad (1.1)$$

where $\Phi(r)$ is related to the gravitational redshift and it is thus the so-called *redshift function*, while $b(r)$ is denoted the *shape function* since it determines the shape of the wormhole.

An alternative way of expressing (1.1) is

$$ds = -e^{2\Phi(r)} dt^2 + dl^2 + r^2(l) \left(d\theta^2 + \sin^2\theta \, d\phi^2\right), \qquad (1.2)$$

where we have set the *proper radial distance*

$$l(r) = \pm \int_{r_0}^{r} \frac{dr}{\sqrt{1 - \frac{b(r)}{r}}}, \qquad (1.3)$$

which is required to be finite everywhere.

Herein $l(r)$ decreases from $l = +\infty$ in the *upper world*, to $l = 0$ at the throat, and then from 0 to $-\infty$ in the *lower world*.

A first *traversability condition* required for the wormhole is to be horizon-free, i.e.

$$g_{tt} = -e^{2\Phi(r)} \neq 0,$$

so that $\Phi(r)$ must also be finite everywhere in the throat.

This is the standard definition.

§1.2. Definitions: the modified metric. We now bring some slight improvement and we re-define the function $b(r)$ as follows

$$b(r) = 1 - \tanh(b_{\text{worm}} + b_{\text{out}}), \qquad (1.4)$$

b_{worm} and b_{out} are here two disjoint *smooth* functions of r that respectively correspond to the wormhole throat, and the *outside* Universe. *Inside* the throat $b_{\text{out}} = 0$, and

$$b(r) = 1 - \tanh b_{\text{worm}} \qquad (1.5)$$

remains the true characteristic of the wormhole solution.

Outside the wormhole, $b_{\text{out}} \gg b_{\text{worm}}$, so we have

$$\tanh\left(b_{\text{worm}} + b_{\text{out}}\right) \to 1\,, \qquad b(r) \to 0\,. \tag{1.6}$$

Therefore, by definition $\Phi(r) \to 0$, and the metric (1.1) reduces to the usual spherically symmetric solution of the Minkowski space

$$ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \tag{1.7}$$

Introducing such an intrinsic breakdown for $b(r)$ enables us to maintain the entire mathematical construction of the wormhole through the new metric:

$$ds^2 = -e^{2\Phi(r)} + \frac{dr^2}{1 - \frac{1-\tanh\left(b_{\text{worm}}+b_{\text{out}}\right)}{r}} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \tag{1.8}$$

Through this modification, we see that the standard wormhole is now surrounded by a *shell*, representing a transient region where the shape function starts to decrease asymptotically from $b(r)$ to 0.

The trick is here obvious:

> This shell does not belong to the inner throat, but is still part of the wormhole geometry together with its physical properties.

We will explain this particular feature for achieving our initial goal in the final course of our theory.

§1.3. The common concept

§1.3.1. The geometric description. The space-time wormhole classically depicted in the Thorne-Morris model is formed with a *static layer* of a particular matter type threading the throat, which was coined by the authors as *exotic matter* (see formal definition below).

Our viewpoint is here different: we consider a dynamical object that actually *produces* the required exotic matter to create the wormhole in which it passes through. Hence it *creates* this space-time distortion, as long as needed for its travel duration.

In the first stage, due to the spherically symmetric nature of the concept and without loss of generality, we restrict the study to the equatorial plane $\theta = \frac{\pi}{2}$, and the interval at $t = const$, so the basic metric reads

$$ds^2 = \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\phi^2 \tag{1.9}$$

still bearing in mind $b(r) = 1 - \tanh\left(b_{\text{worm}} + b_{\text{out}}\right)$.

The coordinate r decreases from $+\infty$ to a minimum value r_0 corresponding to the location (radius) of the wormhole throat, where $b(r_0) = r_0$, and then it increases from r_0 to $+\infty$. The object we have in mind can be best conceived here, as a vertical cylinder centered about the axis z with a radius r_0 that reaches the inner *layer* of the *exotic matter* it carries along.

The reduced metric (1.9) can be embedded into a 3-dimensional Euclidean space, it is written in cylindrical coordinates r, ϕ, z as

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2. \tag{1.10}$$

The embedded surface has equation $z = z(r)$, and the metric of this surface is written

$$ds^2 = \left[1 + \left(\frac{dz}{dr}\right)^2\right] dr^2 + r^2 d\phi^2, \tag{1.11}$$

from these latter two equations, we infer the *slope*

$$\frac{dz}{dr} = \pm \frac{1}{\sqrt{\frac{r}{b(r)} - 1}}. \tag{1.12}$$

In our picture, the inner layer of the *exotic matter* has the minimum radius $r = b(r) = r_0$, at which the embedded surface is vertical (the slope $\frac{dz}{dr} \to \infty$). See Fig. 1.
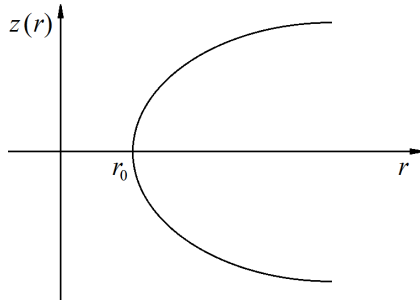

Fig. 1: Cross-section of the created throat.

Far from the *exotic matter layer*, $r \to \infty$, $\frac{dz}{dr} \to 0$, the space is asymptotically flat in accordance with (1.6).

According to (1.3), an object tunnelling through the wormhole has a velocity $v(r)$ passing the throat at r which is measured by a set of

static observers located at this point:

$$v = \frac{dl}{e^{2\Phi(l)}dt} = \pm\frac{dr}{\sqrt{1 - \frac{b}{r}}\ e^{2\Phi(l)}dt}\,. \tag{1.13}$$

Once the travel through the created wormhole is completed, the throat should *flare out* at the end of this creation. Calling $r(z)$, the *embedding function*, its inverse must satisfy the important equation *at* or *near* the distance r_0 to the exotic matter inner layer:

$$\frac{d^2r}{dz^2} = \frac{b - b'r}{2b^2} > 0, \tag{1.14}$$

where the prime denotes the derivative with respect to the radial coordinate r, and within the distance r_0, inspection shows that the form function b should satisfy to $b'(r_0) < 1$.

The relation (1.14) is known as the *flaring out condition*.

§1.3.2. Acceleration gained by an object while traversing the throat. In the present analysis, we will adopt a set of orthonormal basis vectors which we regard as the proper reference frame of a collection of observers remaining at rest in the coordinate system (t, and fixed r, θ, ϕ).

In our case, the orthonormal basis vectors $\hat{e}_a$ are expressed by

$$\left.\begin{aligned}
\hat{e}_t &= e^{-\Phi}e_t \\[4pt]
\hat{e}_r &= \sqrt{1 - \frac{b}{r}}\ e_r \\[4pt]
\hat{e}_\theta &= \frac{e^\theta}{r} \\[4pt]
\hat{e}_\phi &= \frac{e^\phi}{r\sin\theta}
\end{aligned}\right\}. \tag{1.15}$$

With this particular choice, the metric components reduce to the Minkowskian system

$$\hat{e}_a\hat{e}_b = \hat{g}_{ab} = \eta_{ab} = \{-1,\,0,\,0,\,0\}. \tag{1.16}$$

In a general basis, the four-velocity for a static observer is

$$u^a = \frac{dx^a}{d\tau} = \left(u^t,\,0,\,0,\,0\right) = \left[e^{-\Phi(r)},\,0,\,0,\,0\right]. \tag{1.17}$$

From the decomposition of the covariant derivative of u^a, we can extract its four-acceleration

$$a^a = u^a_{;b}\, u^b, \tag{1.18}$$

which reduces, by (1.11), to the following components (denoting the proper time by τ)

$$\left.\begin{aligned} a^t &= 0 \\ a^r &= \{^r_{tt}\}\left(\tfrac{dt}{d\tau}\right)^2 = \Phi'\left(1 - \tfrac{b}{r}\right) \end{aligned}\right\}, \tag{1.19}$$

where a_r is the non-null radial component acceleration required for the observer to follow a geodesic inside the throat (free-fall condition). Herein $\{^a_{bc}\}$ are the Christoffel symbols of the second kind.

We now revert to our orthonormal basis $\hat{e}_a$, and we express the object's proper reference frame in terms of the Special Relativity transformation factor $\gamma = \frac{1}{\sqrt{1-v^2}}$, as

$$\left.\begin{aligned} (\hat{e}_0)_{\mathrm{SR}} &= \gamma\,\hat{e}_t \pm \gamma v\,\hat{e}_r \\ (\hat{e}_1)_{\mathrm{SR}} &= \gamma\,\hat{e}_r \pm \gamma v\,\hat{e}_r \\ (\hat{e}_2)_{\mathrm{SR}} &= e^\theta \\ (\hat{e}_3)_{\mathrm{SR}} &= e^\phi \end{aligned}\right\}. \tag{1.20}$$

Referred to this basis, the object's four-acceleration in its proper reference frame is

$$(\hat{a}^a)_{\mathrm{SR}} = (\hat{u}^a_{;b}\,\hat{u}^b)_{\mathrm{SR}}.$$

In the (t, r, θ, ϕ) coordinate frame, the object moves radially and its acceleration is specialized to t, i.e. $a_t = u_{t;r}\,u^r - \{_{atb}\}u^a u^b$.

Setting $\boldsymbol{a} = |\boldsymbol{a}|\,\hat{e}_\mu$, we note that $a_t = \boldsymbol{a}\,e_t = (a\,\hat{e}_\mu)\,e_t = -\,\gamma v e^\Phi|\boldsymbol{a}|$, and we finally arrive at

$$|\boldsymbol{a}| = \left[\sqrt{1 - \frac{b}{r}}\; e^{-\Phi}(\gamma e^\Phi)'\right]. \tag{1.21}$$

If we now imagine that the object contains some humanoid crew, the acceleration felt by the occupants should obviously not exceed an *earth-like* gravity.

Therefore the expanded acceleration $(\hat{a}^a)_{\mathrm{SR}}$ should satisfy the magnitude constraint ($\oplus$ means the Earth)

$$\boldsymbol{a} \leqslant \boldsymbol{g}_\oplus. \tag{1.22}$$

§1.3.3. The exotic matter. Consider the Einstein tensor expressed with our orthonormal basis $\hat{e}_a$:

$$\hat{G}_{ab} = \hat{R}_{ab} - \frac{1}{2}\,\hat{g}_{ab}\,R\,.$$

In this situation, the components are greatly simplified as

$$\left.\begin{aligned}
\hat{G}_{tt} &= \frac{b'}{r^2}\\[2mm]
\hat{G}_{rr} &= -\frac{b}{r^3} + 2\left(1-\frac{b}{r}\right)\frac{\Phi'}{r}\\[2mm]
\hat{G}_{\theta\theta} &= \left(1-\frac{b}{r}\right)\left[\Phi''+(\Phi')^2-\frac{(b'r-b)\Phi'}{2r(r-b)}-\frac{b'r-b}{2r^2(r-b)}+\frac{\Phi'}{r}\right]\\[2mm]
\hat{G}_{\phi\phi} &= \hat{G}_{\theta\theta}
\end{aligned}\right\}\,. \quad (1.23)$$

If we stick to Birkhoff's theorem [6, p.157], which states that the only vacuum solutions with (static) spherical symmetry is the *Schwartzschild solution*, we are led to introduce a stress-energy tensor T_{ab}. Then, the field equations with a source $\hat{G}_{ab} = 8\pi\hat{T}_{ab}$ induce here the sole non-zero diagonal components of the energy-momentum tensor which classically receive the following (but somewhat arbitrary) physical meanings

$$\hat{T}_{tt} = \rho(r)\,, \quad (1.24)$$

$$\hat{T}_{rr} = -T_{\text{tens}}(r)\,, \quad (1.25)$$

$$\hat{T}_{\theta\theta} = \hat{T}_{\phi\phi} = p(r)\,, \quad (1.26)$$

where $\rho(r)$ is the mass density of the layer, $T_{\text{tens}}(r)$ is the radial tension (or transverse pressure) which is opposed to the radial pressure $p(r)$ ascribed to the mass density ρ of the special layer and which is necessary to sustain the throat. Based on the evident proportionality with the Einstein tensor $\hat{G}_{ab}$, the components of the energy-momentum tensor are

$$\rho(r) = \frac{1}{8\pi}\frac{b'}{r^2}\,, \quad (1.27)$$

$$T_{\text{tens}}(r) = \frac{1}{8\pi}\left[\frac{b}{r^3} - 2\left(1-\frac{b}{r}\right)\frac{\Phi'}{r}\right]\,, \quad (1.28)$$

$$p(r) = \frac{1}{8\pi}\left(1-\frac{b}{r}\right)\left[\Phi''+(\Phi')^2-\frac{(b'r-b)\Phi'}{2r(r-b)}-\frac{b'r-b}{2r^2(r-b)}+\frac{\Phi'}{r}\right]\,. \quad (1.29)$$

From the center of the object to the inner layer of exotic matter, those components reduce to

$$\rho(r_0) = \frac{1}{8\pi} \frac{b'(r_0)}{r_0^2} \,, \tag{1.30}$$

$$T_{\text{tens}}(r_0) = \frac{1}{8\pi r_0^2} \,, \tag{1.31}$$

$$p(r_0) = \frac{1}{8\pi} \frac{1 - b'(r_0)}{2 r_0^2} \left[1 + r_0 \Phi'(r_0) \right] , \tag{1.32}$$

where $b(r_0) = 1 - \tanh(b_{\text{worm}})$.

In the classical wormhole theory, it is customary to introduce the dimensionless function

$$\varsigma = \frac{T_{\text{tens}} - \rho}{|\rho|} \,, \tag{1.33}$$

which is also known as the *exoticity function.*

Using equations (1.27) and (1.28), we find

$$\varsigma = \frac{\frac{b}{r} - b' - 2r \left(1 - \frac{b}{r}\right) \Phi'}{|b'|} \,. \tag{1.34}$$

Taking into account the flaring out condition (1.14), the equation (1.33) takes the form

$$\varsigma = \frac{2b^2}{r\,|b|} \frac{d^2 r}{dz^2} - \frac{2r \left(1 - \frac{b}{r}\right) \Phi'}{|b'|} \,, \tag{1.35}$$

as ρ is finite and so is b', while given the fact that $\left(1 - \frac{b}{r}\right) \Phi' \to 0$ at the throat, we obtain the fundamental relation

$$\varsigma(r_0) = \frac{(T_0)_{\text{tens}} - \rho_0}{|\rho_0|} > 0 \,. \tag{1.36}$$

The restriction

$$(T_0)_{\text{tens}} > \rho_0 \tag{1.37}$$

tells us that the radial tension that is required to sustain the throat, must exceed the layer's mass density.

This is a manifest violation of the *weak energy conditions* (WEC) which states that for any timelike vector u^a, we must have

$$T_{ab}\, u^a u^b \geqslant 0 \,. \tag{1.38}$$

Indeed, consider a radially moving observer inside the throat: for a sufficiently fast velocity, in his basis (1.20), this observer measures an

energy density given by the time component of the stress-energy tensor

$$\hat{T}_{00} = \gamma^2 \hat{T}_{tt} + 2\gamma^2 v^2 \hat{T}_{tr} + \gamma^2 v^2 \hat{T}_{rr} = \gamma^2(\rho_0 - T_{\text{tens}}) + T_{\text{tens}}, \quad (1.39)$$

which is seen *negative*, thus violating the energy conditions (1.38).

If the observer is static, we have the strict condition

$$\rho_0 < 0. \quad (1.40)$$

§1.3.4. The totally exotic matter.

We now define the *averaged null energy condition* (ANEC) which is satisfied along a null curve as

$$\int T_{ab}\, k^a k^b\, d\lambda \geqslant 0, \quad (1.41)$$

where k^a is a null vector and λ is a generic affine parameter.

We then consider an extended type of energy condition involving the *volume integral quantifier*, which is expressed by the two inequalities

$$\int T_{ab}\, u^a u^b\, dV \geqslant 0, \quad \int T_{ab}\, k^a k^b\, dV \geqslant 0, \quad (1.42)$$

where the integral is performed with respect to the proper volume element dV of the exotic matter. With the null vector $\hat{k}^a = (1, 1, 0, 0)$, the expression $\hat{T}_{ab}\, \hat{k}^a \hat{k}^b$ is given by

$$\rho - (T_0)_{\text{tens}} = \frac{1}{8\pi}\left(1 - \frac{b}{r}\right)\left[\ln\left(\frac{e^{2\Phi}}{1 - \frac{b}{r}}\right)\right]'. \quad (1.43)$$

Performing an integration by parts

$$I_{\text{v}} = \int \left[\rho - (T_0)_{\text{tens}}\right] dV =$$

$$= -\frac{1}{8\pi}\left[(r - b)\ln\left(\frac{e^{2\Phi}}{1 - \frac{b}{r}}\right)\right]_{r_0}^{\infty} - \int_{r_0}^{\infty}(1 - b')\left[\ln\left(\frac{e^{2\Phi}}{1 - \frac{b}{r}}\right)\right] dr.$$

At the throat $r_0 = b = [1 - \tanh(b_{\text{worm}})]$ and far from it, the space is asymptotically Euclidean i.e. $\Phi = 0$, so the first part of the right-hand side vanishes, and the (global) energy violation condition is represented by the volume integral

$$I_{\text{v}} = \int \left[\rho - (T_0)_{\text{tens}}\right] dV = -\frac{1}{8\pi}\int_{r_0}^{\infty}(1 - b')\ln\left(\frac{e^{2\Phi}}{1 - \frac{b}{r}}\right) dr. \quad (1.44)$$

If we now want to evaluate the radial *thickness* of the negative energy layer denoted by

$$\Delta = r_{\mathrm{E}} - r_0 \,, \tag{1.45}$$

we just *slice out* a portion of the volume integral (1.44) as

$$I_{\mathrm{V}} = \int \big[\, \rho - (T_0)_{\mathrm{tens}} \,\big]\, dV = -\frac{1}{8\,\pi} \int_{r_0}^{r_{\mathrm{E}}} (1 - b')\ln\left(\frac{e^{2\Phi}}{1 - \frac{b}{r}}\right) dr \,. \tag{1.46}$$

In the equatorial plane representation $\theta = \frac{\pi}{2}$, we will use this volume portion to insert an electromagnetic fluid circulation self-provided by an object (space ship) that creates the wormhole for a limited duration necessary to jump between the upper and the lower worlds.

REMARK: At first glance, one might be tempted to assume an arbitrary small quantities of ANEC-violating matter when $r_{\mathrm{E}} \to r_0$, however further analysis would show that the smaller the amount of exotic matter, the longer the traversable time as measured by external clocks.

Indeed, setting the proper distance $l = -l_1$ in the lower world, and $l = +l_2$ in the upper world, (assuming $\gamma \approx 1$), we let $v = \frac{dl}{d\tau}$, so that $d\tau = \frac{dl}{v}$, and

$$\Delta t = \int_{t_2}^{t_1} dt = \int_{-l_2}^{+l_1} e^{-\Phi(r)}\,\frac{dl}{v} = \int_{r_2}^{r_l} \frac{e^{-\Phi(r)}}{v}\left(1 - \frac{b}{r}\right) dr \,. \tag{1.47}$$

Chapter 2. Achieving the Production of Exotic-Like Matter

§2.1. The electromagnetic field contribution

§2.1.1. The physical stress-energy tensor. Due to the radially symmetric model, Birkhoff's theorem still apply to our modified metric (1.6). However, this theorem does not forbid another type of energy-momentum tensor as a source of the Einstein equations:

$$G_{ab} = R_{ab} - \frac{1}{2}\,g_{ab}\,R = 8\,\pi\,T_{ab} \,.$$

We will then postulate that these equations will possess another type of energy source. Let us first consider the general electrical four-current density

$$j^a = \mu\,u^a, \tag{2.1}$$

where μ is a *time-varying charge density*, coupled to an electromagnetic field characterized by a four-potential A^a.

The resulting energy-momentum tensor for the interacting system is expressed in a general basis as

$$(T^{ab})_{\text{elec}} = \frac{1}{4\pi}\left(\frac{1}{4}\,g^{ab}F_{cd}F^{cd} + F^{ae}F_{e\cdot}^{\cdot b}\right) + g^{ab}j_e\,A^e - j^a A^b, \quad (2.2)$$

from which we extract the energy density as

$$(T^{00})_{\text{elec}} = \frac{1}{4\pi}\left(\frac{1}{4}\,F_{cd}F^{cd} + F^{0e}F_{e\cdot}^{\cdot 0}\right) + j_e\,A^e - j^0 A^0. \quad (2.3)$$

In our specially chosen orthonormal basis (1.15) , the following relations hold

$$(\hat{T}^{00})_{\text{elec}} = \frac{\boldsymbol{E}^2 + \boldsymbol{B}^2}{8\pi} + \boldsymbol{j}\boldsymbol{A}, \quad (2.4)$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are respectively the electric and magnetic field strengths derived from the Maxwell tensor $F_{cd} = \partial_c F_d - \partial_d F_c$.

We suppose that the field potential $A_a(\varphi, \boldsymbol{A})$ is given in the Lorentz gauge, and we set for the three-potential $\boldsymbol{A} = A_\beta$ and for the three-current $\boldsymbol{j} = j_\beta$.

Now, the key feature of our theory consists of implementing the following decomposition equivalence:

$$(\hat{T}_{00})_{\text{elec}} = [(\hat{T}^{00})_{\text{elec}}]_{\text{out}} + [(\hat{T}^{00})_{\text{elec}}]_{\text{worm}} = \frac{(\boldsymbol{E}^2 + \boldsymbol{B}^2)}{8\pi} + \boldsymbol{j}\boldsymbol{A}. \quad (2.5)$$

In this situation, the *positive* free radiative energy density $\frac{\boldsymbol{E}^2 + \boldsymbol{B}^2}{8\pi}$ is *de facto* generated from an electromagnetic field which is located *outside* the wormhole external-layer thickness (see form. 1.45):

$$b_{\text{out}} > r_{\text{E}}. \quad (2.6)$$

This region is obviously the shell precisely defined by b_{out} in the modified metric (1.8).

The finite volume of exotic matter computed as per (1.46) in the equatorial plane representation $\theta = \frac{\pi}{2}$ should here contain a *round-shaped circuit* (e.g. *superconductive medium*) wherein the time-varying current $\boldsymbol{j}$ is circulating. In this case, the interacting term $\boldsymbol{j}\boldsymbol{A}$ is exhibiting its energy density *inside* this volume where the three-current density $\boldsymbol{j}$ must take the *angular* form

$$j_\phi = \mu r\left(\frac{d\phi}{dt}\right) \quad (2.7)$$

with the mean radius $r = \frac{r_{\text{E}} + r_0}{2}$.

§2.1.2. Negative energy density of the interacting term. As described in Maxwell's equations, $\partial_a F^{ab} = 4\pi j^b$, the time component of j^b is just μ and the interacting term $\boldsymbol{j} \boldsymbol{A}$ can be decomposed as

$$
\left.
\begin{aligned}
[(\hat{T}^{00})_{\text{elec}}]_{\text{out}} &= \frac{\boldsymbol{E} \, \nabla\varphi}{4\pi} + \mu\varphi \\
[(\hat{T}^{00})_{\text{elec}}]_{\text{worm}} &= \frac{\left(-\nabla\varphi - \frac{\partial \boldsymbol{A}}{\partial t}\right) \nabla\varphi}{4\pi} + \mu\varphi
\end{aligned}
\right\},
\tag{2.8}
$$

since $E = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial t}$.

In (2.8) the first term in the brackets is always negative. As to the last term, it is made negative when the time-varying scalar charge density μ and the scalar potential φ are 180° out of phase (method reached by the use of *phasors*).

Eventually, in our coordinate basis we need only consider now the equivalence

$$
-\left(\nabla\varphi + \frac{\partial \boldsymbol{A}}{\partial t}\right) + 4\pi\mu\varphi = \frac{b'(r_0)}{2 r_0^2}
\tag{2.9}
$$

always with $b(r_0) = 1 - \tanh(b_{\text{worm}})$ and with $\mu\varphi < 0$.

§2.2. The exotic matter. Reverting now to the energy density expression (1.39) expressed in the basis (1.20)

$$
\hat{T}_{00} = \gamma^2 \hat{T}_{tt} + 2\gamma^2 v^2 \hat{T}_{tr} + \gamma^2 v^2 \hat{T}_{rr} = \gamma^2 (\rho_0 - T_{\text{tens}}) + T_{\text{tens}},
$$

we remember that for a collection of *static observers* the energy density

$$
\hat{T}_{00} = \rho_0 < 0
$$

is seen negative to match the exoticity condition (1.36).

If we then set for the negative matter

$$
[(\hat{T}^{00})_{\text{worm}}]_{\text{elec}} \equiv \rho_0 < 0,
\tag{2.10}
$$

we do have an adequate density mass equivalent.

The substitution (2.10) should not conflict with the other diagonal components of the stress tensor which now correspond to

$$
[(\hat{T}_{rr})_{\text{worm}}]_{\text{elec}} = -(T_{\text{tens}})_{\text{worm}}(r_0),
\tag{2.11}
$$

$$
[(\hat{T}_{\theta\theta})_{\text{worm}}]_{\text{elec}} = [(\hat{T}_{\phi\phi})_{\text{worm}}]_{\text{elec}} = p(r_0).
\tag{2.12}
$$

Note that the mass of the charge has been here discarded in the Einstein field equations since we here assume that the electromagnetic

effects greatly prevail over masses which can thus be neglected. For example, if we use the fundamental leptonic charge which is the electron e, its rest mass is $m_e = 9.1091 \times 10^{-31}$ kg. In this case, for a given three-volume V, we consider e as a point-wise charge, and the charge density μ is then given by

$$\mu = \sum_i e_i \, \delta(x - x^i),$$

where $\delta(x - x^i)$ is the known Dirac function.

All the above reasoning naturally holds for the generalized metric (1.8) when we drop out the constant equatorial plane restriction $\theta = \frac{\pi}{2}$.

Concluding remarks and outlook. We have just here briefly sketched the basic principle of a theory using an electromagnetic field suitably interacting with a time-varying current in order to produce negative energy needed to sustain the space-time wormhole co-generation. Our approach heavily relies on the equivalences (2.10), (2.11) and (2.12) which certainly deserve further scrutiny.

Far reaching traversability and stability conditions are beyond the scope of this paper, as well as additional improved models tending to reduce exotic matter regions. Numerical estimates for the electromagnetic field magnitudes can also be predicted in a separate paper.

However, the story of the space-time wormhole theory does not end here. As soon as 1988, in their famous article [7], Morris, Thorne and Yurtsever have suggested that the traversable wormhole (if feasible) could be used as a time machine. Briefly speaking, they consider two nearby whormoles' mouths labeled 1 and 2. At $t = \tau = 0$, the mouths 1 and 2 are at rest. The mouth 2 is next given an acceleration to reach a near-light velocity, then it reverses its motion to return to its initial (spatial) location. From an exterior observer who measures both mouths, the (proper) time attached to the wormhole 2 is dilated with respect to wormhole 1's time, which has thus *aged* with respect to the second one, when it has come back to its position. As a result, at any later time, if one is tunnelling through the mouth 1, one emerges from the mouth 2 as though one has travelled backward in time. Of course, the reachable past can only *start* from the *date* of creation of the time machine. We cannot however exclude that possible advanced civilizations have already long engineered such a concept so that they are able to travel in a far past perhaps anterior to our human existence.

So much for the principle. There are however a wide range of further complex constraints which are to be overcome.

Starting from *chronal* domains separated from an *achronal* domain by a *future chronology horizon* (special type of the *Cauchy horizon*), Hawking [8] asserted that tunnelling high frequency electromagnetic wave packets would *pile up* against the separation line and finally drive the energy density on the boundary of the hypothetical time machine to infinity, thus destroying this machine at the instant it was created or at least preventing anyone outside of it from entering through it. This is known as the *chronology protection conjecture*: the so-called *closed time curves* (CTCs) at the *chronology horizon* are thus deemed a physical impossibility. This condition is supposed to be required in order to avoid any time paradox, a cliché which has since been strictly ruled out by several physicists (see for instance Klinkhammer, 1992 private communication).

Quite recently, several authors [9, 10] have thus challenged these statements, and the physicist Li-Xin Li [11] has even *rejected* the Hawking conjecture. Without going into sound technicalities, it suffices to know that the vacuum metric fluctuations (close to the Planck length scale) produce unwanted effects on the *defocusing* exerted by the wormhole on the amplitude of any classical high-frequency waves propagating along a null geodesic (light) following the inner *walls* of the wormhole axis, thus eventually causing it to collapse.

Analyzing the total cross sections for various particles' pair collisions or formations (see, for example, form. 94.6 in [12], and related formal derivation), Li has strictly demonstrated that by inserting an opaque absorption material with a definite transmission coefficient including any of these cross sections into the inner wall of the wormhole, the metric fluctuations tend to zero, leading to a stable Lorentzian wormhole.

Nevertheless, and although some real positive progresses are increasingly emerging, we clearly see that a deeper amount of research work remains to be carried out, before feasible solutions can eventually be found.

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